A Theory of Demand Shocks

Guido Lorenzoni*

November 2008

Abstract

This paper presents a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The public signal gives rise to "noise shocks," which have the features of aggregate demand shocks: they increase output, employment and inflation in the short run and have no effects in the long run. Numerical examples suggest that the model can generate sizeable amounts of noise-driven volatility.

JEL Codes: E32, D58, D83

Keywords: Consumer confidence, aggregate demand shocks, business cycles, imperfect information.

---

*Department of Economics, Massachusetts Institute of Technology, 50 Memorial Drive, Cambridge, MA 02142 and National Bureau of Economic Research. Email: glorenzo@mit.edu. A previous version of this paper circulated under the title: "Imperfect Information, Consumers' Expectations, and Business Cycles." I thank for useful comments Mark Gertler, three anonymous referees, Marios Angeletos, Robert Barsky, Olivier Blanchard, Ricardo Caballero, Michele Cavallo, Larry Christiano, Veronica Guerrieri, Christian Hellwig, Steve Morris, Nancy Stokey, Laura Veldkamp, Iván Werning, Mike Woodford and seminar participants at MIT, University of Virginia, Ohio State University, UCLA, Yale, NYU, Ente Einaudi-Rome, the Minnesota Workshop in Macro Theory, Columbia University, Boston College, University of Michigan, AEA Meetings (Philadelphia), University of Cambridge, LSE, UCL, NBER EFG, Northwestern, Boston University. I thank the Federal Reserve Bank of Chicago for its hospitality during part of this project.

The idea that changes in consumer expectations have a causal effect on cyclical fluctuations is widespread in business discussions and policy debates. Recent dynamic stochastic general equilibrium models of the business cycle include a large number of shocks (to technology, monetary policy, preferences, etc.), but typically do not include expectational shocks as independent drivers of short-run fluctuations.[1] This paper explores the idea of expectation-driven cycles, looking at a model where technology determines equilibrium output in the long run, but consumers only observe noisy signals about technology in the short run. The presence of noisy signals produces expectational errors. This paper studies the role of these expectational errors in generating volatility at business cycle frequencies.

The model is based on two basic ingredients. First, consumers take time to recognize permanent changes in aggregate fundamentals. Although they may have good information on the current state of the individual firm or sector where they operate, they only have limited information regarding the long-run determinants of aggregate activity. Second, consumers have access to public information which is relevant to estimate long-run productivity. This includes news about technological innovations, publicly released macroeconomic and sectoral statistics, financial market prices, and public statements by policy-makers. However, these signals only provide a noisy forecast of the long-run effects of technological innovations. This opens the door to "noise shocks," which induce consumers to temporarily overestimate or underestimate the productive capacity of the economy.

In this paper, I derive the model's implications for the aggregate effects of actual technology shocks and noise shocks. In particular, the theory imposes restrictions on the relative responses of output, employment, and inflation following the two types of shocks and it places an upper bound on the amount of short-run volatility that noise shocks can generate, for a given level of fundamental volatility.

The analysis is based on a standard new Keynesian model where I introduce both aggregate and idiosyncratic productivity shocks. The average level of productivity in the economy follows a random walk. However, agents cannot observe average productivity directly. They can only observe the productivity level in their own sector, which

[1] Recent exceptions include Jean-Pierre Danthine et al. (1998), Paul Beaudry and Franck Portier (2004), Nir Jaimovich and Sergio Rebelo (2006). The relation with these papers is discussed below.

has a temporary idiosyncratic component, and a noisy public signal regarding average productivity. They also observe prices and quantities which provide endogenous sources of information.

In this environment, a positive technology shock leads to a gradual adjustment in output to its new long run level, and to a temporary fall in employment and inflation. On the other hand, a positive noise shock leads to a temporary increase in output, employment and inflation.

The mechanism behind these effects is essentially based on the consumers' Euler equation. Current consumption depends positively on expected future consumption and negatively on the expected real interest rate. In equilibrium, agents expect future consumption to converge to a level determined by permanent changes in technology. Due to nominal rigidities, the real interest rate responds sluggishly to shocks. Therefore, consumption is mainly driven, in the short run, by changes in expectations about permanent productivity. After a technology shock, expectations respond less than one-for-one to the change in average productivity, given that consumers only observe noisy signals about it. Demand lags behind actual productivity, leading to a temporary fall in employment and to a deflationary pressure. A noise shock, on the other hand, has the features of a pure "aggregate demand shock." As consumers temporarily overstate the economy's productive capacity, demand increases while productivity is unchanged. This generates a temporary increase in employment and inflation.

To present my argument, I consider first a simple representative agent version of the model, without dispersed information, where the idiosyncratic temporary productivity shock is replaced by an aggregate temporary productivity shock. This basic model can be solved analytically and provides the essential intuition for the full model with dispersed information. However, this version of the model requires large temporary productivity shocks to prevent agents from learning long-run productivity too quickly. Dispersed information provides a more realistic way to slow down aggregate learning, so that noise shocks can have sizeable and persistent effects in the short run. When I turn to the model with dispersed information, I resort to numerical simulations. The computation of the model poses some technical challenges, reflecting the infinite regress problem that arises when agents "forecast the forecasts of others," as in Robert Townsend (1983). To address this problem, I develop a method of indeterminate coefficients with a truncated state space.

Numerical simulations are used both to analyze the qualitative predictions of the model and for a simple quantitative exercise. Namely, I evaluate the size of the demand disturbances generated by noise shocks in the model, under different parametrizations, and compare them with the demand disturbances in a simple bivariate VAR of output and employment, similar to Jordi Galí (1999). These simulations suggest that noise shocks may be able to produce a sizeable fraction of observed demand-side volatility.

A novel element of a business cycle model based on learning and noise is that the choice of variance parameters have complex, non-monotonic effects on the model's dynamics. This is because variance parameters affect not only the volatility of the shocks, but also the inference problem of the agents. In particular, a crucial parameter is the variance of the noise in the public signal. When this variance is either too small or too large, noise shocks generate small amounts of short-run volatility. In the first case, public signals are very precise and the economy converges immediately to the full information equilibrium. In the second case, public signals are very imprecise and private agents tend to disregard them in their inference. Therefore, intermediate levels of noise variance are required to generate sizeable amounts of short-run volatility.

Finally, I present a simple test which lends support to a central prediction of the model. According to the model, average expectations tend to underreact following an actual technology shock and to overreact following a noise shock. The reason is that, in the first case, consumers are optimistic, but actual productivity is even better than their expectations. In this case, producers tend to lower prices, leading to a stronger output response. In the second case, consumers are also optimistic, but actual productivity has not changed. Then producers tend to increase prices, leading to a weaker output response. To test this hypothesis I look at two measures of expectations about aggregate activity, from the Survey of Professional Forecasters and from the Michigan Consumer Sentiment Survey, and I look at their responses to identified technology and non-technology shocks from a bivariate VAR. In both cases, non-technology shocks tend to have a relatively larger effect on expectations than technology shocks, although the difference is significant only when using the Survey of Professional Forecasters data.

The idea that expectations and expectational errors play a relevant role in explaining business cycles goes back, at least, to Arthur C. Pigou (1929) and John M. Keynes (1936).

This idea has received renewed attention in a number of papers, including Danthine et al. (1998), Beaudry and Portier (2004, 2006), Jaimovich and Rebelo (2006).[2] These papers emphasize the distinction between shocks to current and future productivity.[3] My paper, instead, emphasizes the difference between fundamental and noise shocks.

The signal-extraction problem faced by consumers connects this paper to a number of papers in which consumers observe noisy signals of underlying shocks. The closest are Antulio Bomfim (2001) and Rochelle M. Edge et al. (2007), who introduce noisy observations of TFP in standard Real Business Cycle models. In particular, Bomfim (2001) focuses on the effects of measurement error in current TFP, which is similar to the noise shock in my model.[4] The main differences are that my paper features nominal rigidities and dispersed information and that it focuses on the conditional behavior of output, hours and inflation after noise shocks vs fundamental shocks.

A few papers have tried to measure empirically the effects of expectational shocks due to noisy information, focusing on specific signals available in real time to the private sector. In particular, Seonghwan Oh and Michael Waldman (1990) focus on the measurement error present in the early release of the leading economic indicators and show that shocks to this error term have sizeable positive effects on aggregate activity. José V. Rodriguez Mora and Paul Schulstad (2007) show that aggregate consumption responds more to early public announcements regarding aggregate GDP than to actual movements in GDP, as measured by the revised GDP series. Both results are consistent with the approach in this paper.

Recent work on estimated dynamic stochastic general equilibrium models has identified intertemporal disturbances affecting the consumers' Euler equation as important drivers of the business cycle (Giorgio Primiceri et al., 2006). These intertemporal disturbances are somehow treated as a residual, as they are attributed to shocks to intertemporal preferences. In this paper, I provide an alternative foundation for shocks to the consumers' Euler equations, as shocks coming from changes in average expectations

[2]Olivier J. Blanchard (1993) and John H. Cochrane (1994) are early papers that point attention to endogenous movements in consumption as a driving force behind cyclical fluctuation.

[3]Simon Gilchrist and John V. Lehay (2002) and Lawrence J. Christiano et al. (2006) explore the effects of shocks to future productivity on the joint behavior of aggregate activity and asset prices, and study the optimal response of monetary policy to these shocks.

[4]This formulation goes back to the original informational setup in Finn E. Kydland and Edward C. Prescott (1982). The signal-extraction problem played a minor role in their analysis and was later discarded in the development of Real Business Cycle models.

about long-run fundamentals.

The modelling approach in this paper is related to various strands of literature. The idea that imperfect information can cause sluggish adjustment in economic variables and generate fluctuations driven by expectational errors, goes back to Edmund S. Phelps (1969) and Robert E. Lucas (1972). More recently, Michael Woodford (2002), N. Gregory Mankiw and Ricardo Reis (2002), and Christopher A. Sims (2003), have renewed attention to imperfect information and limited information processing as sources of inertial behavior.[5] Finally, a rich literature, starting with Stephen Morris and Hyun Song Shin (2002), has emphasized that, in environments with imperfect information, public sources of information can cause persistent deviations of economic variables from their fundamental values.[6] This paper puts together ideas from these literatures to build a model of the cycle based on noisy learning. [7]

Finally, the paper is related to the literature on optimal monetary policy with uncertain fundamentals.[8] That literature focuses on the central bank's uncertainty regarding these fundamentals, while here I focus on the private sector's uncertainty.

The paper is organized as follows. Section I presents the representative agent model with common information which is used to illustrate the basic mechanism of the paper. In Section II, I introduce the model with dispersed information. In Section III, I present numerical simulations of the model. In Section IV, I present the test based on survey data. Section V concludes.

[5]See also Fabrice Collard and Harris Dellas (2004), Giuseppe Moscarini (2004), Christian Hellwig (2005), Klaus Adam (2006), Philippe Bacchetta and Eric Van Wincoop (2006), Yulei Luo (2006), Bartosz Maćkowiak and Mirko Wiederholt (2006), Reis (2006), Laura Veldkamp and Stijn Van Nieuwerburgh (2006), Fabio Milani (2007), Kristoffer Nimark (2008).

[6]See Hellwig (2002), George-Marios Angeletos and Alessandro Pavan (2004), Jeffery Amato et al. (2005), Franklin Allen et al. (2006).

[7]Takuji Kawamoto (2004) looks at the effect of technology shocks in an environment with imperfect information. His analysis does not feature noise shocks and focuses on the gradual adjustment of output after a technology shock. He independently derives the result that, under imperfect information, technology shocks lead to a fall in employment.

[8]See Kosuke Aoki (2003), Athanasios Orphanides (2003), Reis (2003), Lars E.O. Svensson and Woodford (2003, 2005), Andrea Tambalotti (2003).

I. A basic model

A. *Setup*

Let me begin by considering a simple representative agent model with common information, which illustrates the basic mechanism of the paper. The model is a standard new Keynesian model with monopolistic competition and price setting *à la* Guillermo Calvo (1983). In this environment, I introduce temporary and permanent aggregate technology shocks and assume that agents cannot distinguish the two shocks and receive a noisy public signal regarding the permanent shock. I then analyze the economy's dynamic behavior, focusing on the effect of the "noise shock" which corresponds to the noise component in the public signal.

Preferences and technology. The preferences of the representative consumer are given by

$$E\sum_{t=0}^{\infty}\beta^t U\left(C_t,N_t\right),$$

with

$$U\left(C_t,N_t\right)=\log C_t-\frac{1}{1+\zeta}N_t^{1+\zeta},$$

where N_t are hours worked and C_t is a composite consumption good given by

$$C_t=\left(\int_0^1 C_{j,t}^{\frac{\gamma-1}{\gamma}}dj\right)^{\frac{\gamma}{\gamma-1}},$$

$C_{j,t}$ is the consumption of good j in period t, and $\gamma>1$ is the elasticity of substitution among goods. Each good $j\in[0,1]$ is produced by a single monopolistic firm, which has access to the linear production function

$$Y_{j,t}=A_tN_{j,t}. \tag{1}$$

Uncertainty. The only source of exogenous uncertainty is the productivity parameter A_t. Let $a_t=\log A_t$. From now on, a lowercase variable will denote the log of the corresponding uppercase variable. Productivity has a permanent component, x_t, and a temporary component, η_t,

$$a_t=x_t+\eta_t, \tag{2}$$

where η_t is an i.i.d. shock, normal, with zero mean and variance σ_η^2, and x_t is a random walk process given by

$$x_t = x_{t-1} + \epsilon_t, \tag{3}$$

where ϵ_t is i.i.d., normal, with zero mean and variance σ_ϵ^2. Each period all agents in the economy observe current productivity a_t and the noisy signal s_t regarding the permanent component of the productivity process, given by

$$s_t = x_t + e_t, \tag{4}$$

where e_t is i.i.d., normal, with zero mean and variance σ_e^2. The three shocks η_t, ϵ_t and e_t are mutually independent.

The noise term e_t in the signal s_t plays two roles: it prevents the agents from perfectly identifying permanent innovations to technology *and* it generates an independent source of variation in the agents' beliefs regarding x_t. As I will show below, both roles are relevant in determining the economy's cyclical behavior.

Consumers. I consider a simple "cashless" environment where consumers have access to a nominal one-period bond which trades at the price Q_t. The consumer's budget constraint is

$$Q_t B_{t+1} + \int_0^1 P_{j,t} C_{j,t} dj = B_t + W_t N_t + \int_0^1 \Pi_{j,t} dj, \tag{5}$$

where B_t are nominal bonds' holdings, $P_{j,t}$ is the price of good j, W_t is the nominal wage rate, and $\Pi_{j,t}$ are the profits of firm j. In equilibrium consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (5) and a standard no-Ponzi-game condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires that $B_t = 0$.

From consumers' optimization it follows that the demand for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} C_t, \tag{6}$$

where P_t is the price index

$$P_t = \left(\int_0^1 P_{j,t}^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}}. \tag{7}$$

Firms. Firms are allowed to reset prices only at random time intervals. Each period, a firm is allowed to reset its price with probability $1-\theta$ and must keep the price unchanged with probability θ. Firms hire labor on a competitive labor market at the wage W_t, which is fully flexible.

The firm's objective is to maximize the expected present value of its profits. Since the firms are owned by the consumers, this present value is computed using the stochastic discount factor $Q_{t+\tau|t} = \beta^\tau \left(C_{t+\tau}/C_t\right)^{-1}$. Let P_t^* denote the optimal price for a firm who can adjust its price at time t. This firm maximizes

$$E_t \sum_{\tau=0}^{\infty} \theta^\tau Q_{t+\tau|t} \left[P_{j,t+\tau} Y_{j,t+\tau} - W_{t+\tau} N_{j,t+\tau}\right],$$

subject to $P_{j,t+\tau} = P_t^*$, the technological constraint (1), and the demand relation (6). The firm takes as given the stochastic processes for P_t, C_t, and W_t, and the stochastic discount factor $Q_{t+\tau|t}$.

Aggregate real output is defined as nominal output divided by the price index P_t,

$$Y_t \equiv \frac{\int_0^1 P_{j,t} Y_{j,t} dj}{P_t}.$$

Substituting (6) and (7) on the right-hand side, it follows that $Y_t = C_t$, so aggregate output is equal to aggregate consumption. Inflation is defined as the change in the log of the price index P_t, that is,

$$\pi_t \equiv p_t - p_{t-1}.$$

Monetary policy. To complete the description of the environment, I need to specify a monetary policy rule. The central bank sets the short-term nominal interest rate, i.e., it sets the price of the one-period nominal bond, Q_t. Letting $i_t = -\log Q_t$, I can describe monetary policy in terms of choosing i_t each period. For simplicity, I focus on a simple rule which responds only to current inflation

$$i_t = i^* + \phi \pi_t, \tag{8}$$

where $i^* = -\log \beta$ and ϕ is a constant coefficient chosen by the monetary authority.

B. *Equilibrium*

Following standard steps, the consumers' and the firms' optimality conditions and the market clearing conditions can be log-linearized and transformed so as to obtain two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium.[9]

In particular, the consumer's Euler equation and goods market clearing give the relation[10]

$$y_t = E_t [y_{t+1}] - i_t + E_t [\pi_{t+1}] . \tag{9}$$

The firm's optimal pricing condition can be manipulated so as to obtain

$$\pi_t = \lambda (w_t - p_t - a_t) + \beta E_t [\pi_{t+1}] , \tag{10}$$

where $\lambda \equiv (1 - \theta)(1 - \beta\theta)/\theta$ is a constant parameter. The first term on the right-hand side reflects the effect of real marginal costs, captured by $w_t - p_t - a_t$, on the desired price-target of the firms who can adjust prices. Substituting in (10) the consumer's optimality condition for labor supply, $w_t - p_t - y_t = \zeta n_t$, and the labor market clearing condition, $n_t = y_t - a_t$, one obtains

$$\pi_t = \kappa (y_t - a_t) + \beta E_t [\pi_{t+1}] , \tag{11}$$

where $\kappa \equiv \lambda (1 + \zeta)$.

Equations (9) and (11), together with the monetary rule (8), can be used to derive the equilibrium dynamics of y_t and π_t. Let $x_{t|t}$ denote the agents' expectation regarding x_t based on their information at date t, that is

$$x_{t|t} \equiv E_t [x_t] .$$

To characterize the equilibrium, let me begin with the following conjectures regarding

[9] See Galí (2007), Chapter 3.

[10] From now on, throughout the paper, I will omit constant terms in linear equations, whenever confusion is not possible.

the one-step-ahead forecasts of output and inflation:

$$E_t[y_{t+1}] = E_t[x_t], \tag{12}$$
$$E_t[\pi_{t+1}] = 0. \tag{13}$$

Substituting these conjectures and the monetary policy rule (8) in (9) and (11) gives

$$\begin{aligned} y_t &= x_{t|t} - \phi\pi_t, \\ \pi_t &= \kappa\left(y_t - a_t\right). \end{aligned}$$

The first equation reflects the fact that current consumption, and hence current output, depend positively on the agents' expectations regarding the permanent component of technology and negatively on current inflation, which tends to raise the nominal interest rate and, given (13), the real interest rate. The second equation shows that current inflation depends positively on the difference between current output and "natural output," which is equal to a_t.[11] Rearranging, I obtain

$$y_t = \frac{1}{1+\phi\kappa}x_{t|t} + \frac{\phi\kappa}{1+\phi\kappa}a_t, \tag{14}$$
$$\pi_t = \frac{\kappa}{1+\phi\kappa}\left(x_{t|t} - a_t\right). \tag{15}$$

Equation (14) shows that realized output is a weighted average of productivity and of the agents' expectation about permanent productivity, $x_{t|t}$. The relative weights depend on the parameters κ and ϕ, which capture, respectively, the importance of nominal rigidities in the model and the responsiveness of monetary policy to inflation. I will return to the role of these parameters below. Taking expectations on both sides of (14) and (15) at time $t-1$ and using the fact that x_t is a random walk, confirms the initial conjectures (12) and (13).

[11] Natural output is defined as the output level that arises under flexible prices. Since prices are flexible in the limit case where $\theta \to 0$ and $\kappa \to \infty$, expression (14), below, shows that indeed natural output is equal to a_t.

C. *Productivity shocks and noise shocks*

While expressions (14) and (15) provide a compact characterization of the equilibrium behavior of output and inflation, to fully characterize the economy's response to the underlying shocks $(\epsilon_t, e_t, \eta_t)$ it is necessary to derive an explicit expression for $x_{t|t}$. Each period, the agents observe two noisy signals regarding the aggregate state x_t: current productivity a_t and the current signal s_t. Applying standard Kalman filtering techniques the dynamics of $x_{t|t}$ are given by

$$x_{t|t} = \rho x_{t-1|t-1} + (1-\rho)\left(\delta s_t + (1-\delta)\, a_t\right), \tag{16}$$

where ρ and δ are scalars in $(0,1)$, which depend on the variance parameters σ_ϵ^2, σ_e^2 and σ_η^2.[12] In particular, the parameter ρ is increasing in σ_e^2 and σ_η^2, given that, when these variances are larger, s_t and a_t are less precise signals of x_t and agents take longer to adjust their expectation $x_{t|t}$ to the true value of x_t. The parameter δ, instead, depends on the ratio σ_e^2/σ_η^2, that is, on the relative precision of the two signals. The more precise is s_t, relative to a_t, the larger the value of δ.

Now it is possible to study the effect of the three underlying shocks ϵ_t, e_t, and η_t, by deriving the impulse response functions of y_t, n_t, and π_t. Let me begin by considering a permanent productivity shock $\epsilon_t = 1$. The response of realized productivity, $a_{t+\tau}$, τ periods after the shock, is 1 for all $\tau \geq 0$. The response of the agents' expectation $x_{t+\tau|t+\tau}$ is equal to $\sum_{k=0}^{\tau} \rho^k (1-\rho) = 1 - \rho^{\tau+1}$. To derive this expression, iterate (16) forward and notice that, after the shock, both $s_{t+\tau}$ and $a_{t+\tau}$ increase permanently. Substituting

[12] The expressions for ρ and δ are

$$\begin{aligned} \rho &= \frac{1/\sigma_x^2}{1/\sigma_x^2 + 1/\sigma_\eta^2 + 1/\sigma_e^2} \\ \delta &= \frac{1/\sigma_e^2}{1/\sigma_e^2 + 1/\sigma_\eta^2}, \end{aligned}$$

where $\sigma_x^2 \equiv Var_{t-1}[x_t]$ is the solution to the Riccati equation

$$\sigma_x^2 = \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_e^2}\right)^{-1} + \sigma_\epsilon^2.$$

I assume that the prior of the agents at time $t=0$ is $N\left(0, \sigma_x^2\right)$ so that the agents' learning problem has already reached its steady state and the coefficients ρ and δ are time invariant.

in (14) yields the following impulse response function for output

$$\frac{dy_{t+\tau}}{d\epsilon_t} = 1 - \frac{\rho^{\tau+1}}{1+\phi\kappa} \in (0,1) .$$

The output response to a permanent productivity shock is positive, grows with τ, and converges to 1 as $\tau \to \infty$. Since $n_t = y_t - a_t$, the response of employment is

$$\frac{dn_{t+\tau}}{d\epsilon_t} = -\frac{\rho^{\tau+1}}{1+\phi\kappa} < 0,$$

giving a temporary negative response of employment which dies out as $\tau \to \infty$. Using (15), also inflation displays a temporary negative response, with an impulse response function given by

$$\frac{d\pi_{t+\tau}}{d\epsilon_t} = -\kappa\frac{\rho^{\tau+1}}{1+\phi\kappa} < 0.$$

The intuition behind these responses is that agents are not able to immediately identify the permanent technology shock. Therefore, the expectation $x_{t|t}$ initially underreacts relative to the actual change in underlying productivity. This implies that consumers' demand, and thus output, catch up only gradually with the increased productivity of the economy. Along the transition path, firms tend to lower prices as they face lower marginal costs, and employment falls temporarily.

Let me turn to the noise shock e_t, which is a pure shock to expectations and does not affect productivity. Again, it is useful to first derive the responses of $a_{t+\tau}$ and $x_{t+\tau|t+\tau}$ to $e_t = 1$. The response of $a_{t+\tau}$ is clearly zero for all $\tau \geq 0$. The filtering equation (16) shows that the response of $x_{t+\tau|t+\tau}$ is now given by $\rho^\tau (1-\rho)\,\delta$ for $\tau \geq 0$. Using (14) it then follows that the response of output is

$$\frac{dy_{t+\tau}}{de_t} = \rho^\tau \frac{(1-\rho)\,\delta}{1+\phi\kappa} > 0,$$

the response of employment is

$$\frac{dn_{t+\tau}}{de_t} = \rho^\tau \frac{(1-\rho)\,\delta}{1+\phi\kappa} > 0,$$

and, using (15), the response of inflation is

$$\frac{d\pi_{t+\tau}}{de_t} = \rho^\tau \frac{\kappa (1-\rho) \delta}{1+\phi\kappa} > 0.$$

Therefore, output, employment, and inflation all increase in the short run and then revert to their initial values as $\tau \to \infty$.

The response to a temporary shock η_t is more complex, because in the first period the shock affects both the agents' beliefs and realized productivity, while in the following periods it only affects the agents' beliefs. Proceeding as in the previous cases, it is easy to show that the first period responses of output, employment, and inflation are, respectively,

$$\begin{aligned}
\frac{dy_t}{d\eta_t} &= \frac{(1-\rho)(1-\delta)+\phi\kappa}{1+\phi\kappa} > 0, \\
\frac{dn_t}{d\eta_t} &= \frac{(1-\rho)(1-\delta)-1}{1+\phi\kappa} < 0, \\
\frac{d\pi_t}{d\eta_t} &= \frac{\kappa\left((1-\rho)(1-\delta)-1\right)}{1+\phi\kappa} < 0.
\end{aligned}$$

In the following periods, the responses are all positive and equal, respectively, to

$$\begin{aligned}
\frac{dy_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{(1-\rho)(1-\delta)}{1+\phi\kappa}, \\
\frac{dn_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{(1-\rho)(1-\delta)}{1+\phi\kappa}, \\
\frac{d\pi_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{\kappa (1-\rho)(1-\delta)}{1+\phi\kappa}.
\end{aligned}$$

After the effect on productivity has vanished, the effect of η_t is analogous to that of a noise shock, since it only affects agents' expectations.

This simple model suggests that a setup where agents learn about long-run changes in productivity delivers interesting implications about the conditional correlations of output, inflation, and employment, following different shocks. In particular, the impulse responses derived above suggest that the noise shock has the flavor of an aggregate demand shock in traditional Keynesian models.

D. *Remarks*

Inspecting (14) immediately reveals that the two crucial parameters for the model's dynamics are κ, reflecting the importance of nominal rigidities in the model,[13] and ϕ, reflecting the monetary policy response to inflation. When either κ or ϕ are larger, equilibrium output tends to be closer to current productivity. In the flexible price limit (with $\theta \to 0$ and $\kappa \to \infty$), the long-run expectations of consumers only determine the real interest rate but have no impact on equilibrium output. This emphasizes that the role of consumer expectations on equilibrium output is very different depending on the degree of price stickiness. Nominal rigidities mute the response of the real interest rate and imply that shifts in consumers' expectations are translated into changes in current output. With flexible prices, instead, changes in expectations are completely absorbed by the real rate.

On the monetary policy side, as ϕ goes to infinity the equilibrium converges to the equilibrium of a flexible price economy irrespective of the value of κ. In that case, the central bank adjusts the nominal interest rate so as to mimic the movements in the real rate in the flexible price benchmark.[14] Notice that $\phi \to \infty$ corresponds to the optimal monetary policy in this environment, as it delivers both zero inflation and a zero output gap. In this sense, the demand shocks identified above are the result of a suboptimal policy rule. Extending the model, there are a number of reasons why optimal monetary policy may not be able to mimic the flexible price benchmark in this type of environment. For example, one could introduce mark-up shocks, affecting the pricing equation, and assume that the monetary authority can only observe y_t and π_t. In this case, the monetary authority would not be able to identify the values of a_t and $x_{t|t}$ (which are needed to compute the "natural rate") and would have to base its actions on its best estimates of these variables. The analysis of optimal monetary policy in such an environment is outside the scope of this paper.[15]

Notice that in the model there is a non-trivial relation between the variances $\sigma_\epsilon^2, \sigma_e^2$,

[13] Notice that κ is decreasing in θ, for given β and ζ.

[14] Substituting (15) into (8) shows that as $\phi \to \infty$ the nominal interest rate, and thus the real interest rate, converge to $x_{t|t} - a_t$.

[15] See Aoki (2003) and Svensson and Woodford (2003) for related exercises in environments where the private sector has full information. In Lorenzoni (2007), I analyze optimal monetary policy in an environment with dispersed information analogous to that in Section II.

and σ_η^2 and the output volatility generated, respectively, by the three shocks. In particular, consider the short-run (one period) output volatility due to noise shocks, which is equal to

$$\left(\frac{(1-\rho)\,\delta}{1+\phi\kappa}\right)^2 \sigma_e^2. \tag{17}$$

Notice that, as σ_e^2 approaches 0 the value of $(1-\rho)\,\delta$ converges to 1, since in the limit the signal s_t conveys perfect information about x_t. When instead σ_e^2 goes to ∞, the expression $(1-\rho)\,\delta$ goes to 0, as the signal becomes completely uninformative.[16] In both cases, the expression in (17) goes to 0.[17] That is, when the signal is too precise or too imprecise, noise shocks tend to generate small levels of output volatility. In order for noise shocks to have a relevant cyclical effect, one needs to consider intermediate values for σ_e^2, so that agents put some weight on the signal s_t, while, at the same time, the noise e_t is sufficiently volatile. This non-monotonic relation between the variance of the noise shocks and the output volatility they generate is a peculiar feature of a learning model of the business cycle. I will return to this point in the numerical analysis of the model with dispersed information, in Section B.

E. *Towards imperfect information*

The objective of this paper is to build a model where demand disturbances are generated by informational shocks on aggregate productivity. By "demand disturbances" I mean disturbances which generate positive comovement between output, employment and inflation. The noise shocks in the simple model above have these features. However, in order to generate a sizeable amount of volatility, they need to be accompanied by rel-

[16]Using the expressions in footnote 12, it is easy to show that

$$\lim_{\sigma_e^2 \to 0} \rho = 0, \lim_{\sigma_e^2 \to 0} \delta = 1,$$

and

$$\lim_{\sigma_e^2 \to \infty} \rho = \frac{2\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\epsilon^2 + \sqrt{\left(\sigma_\epsilon^2\right)^2 + 4\sigma_\eta^2\sigma_\epsilon^2}}, \lim_{\sigma_e^2 \to \infty} \delta = 0.$$

[17]To prove this statement, in the second case, notice that

$$\lim_{\sigma_e^2 \to \infty} \left((1-\rho)\,\delta\right)^2 \sigma_e^2 = \lim_{\sigma_e^2 \to \infty} \frac{1/\sigma_e^2}{\left(1/\sigma_x^2 + 1/\sigma_\eta^2 + 1/\sigma_e^2\right)^2} = 0.$$

atively large temporary shocks to productivity, that is, by a relatively large value of σ_η^2. From an analytical point of view, notice that, as σ_η^2 goes to zero δ goes to zero, agents only use current productivity to forecast x_t, and the output volatility due to noise shocks in (17) goes to zero.[18] From a quantitative point of view, I have simulated the model above, using parameters consistent with those in Section III below. These simulations show that in order to obtain sizeable and persistent effects of noise shocks on output (in the range of those obtained in B) requires values of σ_η at least ten times larger than σ_ϵ. These values not only appear unrealistic, but, in a way, defeat the purpose of the exercise: in order to introduce sizeable demand disturbances requires the introduction of much larger "supply disturbances," that is, disturbances which generate a negative correlation between output and inflation and between output and employment in the short run. In fact, numerical simulations show that these values of σ_η lead to negative unconditional correlations of these variables at business cycle frequencies.[19]

The role of the temporary shock η_t in the model of this section was essentially to add noise to the observation of x_t by the representative agent, so as to induce sluggish adjustment in expectations. A realistic alternative is to introduce idiosyncratic productivity shocks and assume that agents can only directly observe productivity in their own sector, which is a noisy signal about average productivity in the economy. This approach is appealing for several reasons. First, the introduction of large idiosyncratic shocks seems more realistic, as the available evidence points to the presence of large firm-level volatility, relative to aggregate volatility.[20] Second, this approach is consistent with the presence of considerable dispersion in expectations about macroeconomic aggregates, both in measures of consumer sentiment and in the survey of professional forecasters.[21] Third, given that idiosyncratic productivity shocks cancel out in the aggregate, they do not contribute to generate a negative correlation between output and employment and between output and inflation, that is, they allow me to introduce demand shocks without, at the same time, having to introduce large supply shocks.

[18] This follows from the expressions in footnote 12.

[19] This is due to the fact that temporary productivity shocks are associated to a fall in employment. See the related discussion at the end of Section 4.2.

[20] For example, Diego Comin and Thomas Philippon (2005) show that firm-level sales volatility is an order of magnitude larger than the volatility of aggregate output. See also Francesco Franco and Philippon (2007).

[21] The presence of this dispersion in expectations has been recently emphasized by Mankiw et al. (2003) and Olivier Coibion and Yuriy Gorodnichenko (2008) who focus on inflation expectations.

Therefore, in the following section, I modify the model by introducing idiosyncratic productivity shocks and dispersed information. In particular, the temporary shock η_t will be replaced by idiosyncratic, sector-specific temporary productivity shocks $\eta_{l,t}$. I will then assume that agents can only observe productivity in their own sector and noisy price and quantity signals about the aggregate economy.

II. The model with dispersed information

A. *Setup*

I now turn to the full model with heterogeneity and dispersed information. Consumers are located in a continuum of islands, indexed by $l \in [0,1]$. Each island is analogous to the economy described in the previous section, with a representative consumer who owns a continuum of price-setting firms producing differentiated goods indexed by $j \in [0,1]$. However, now islands are characterized by different productivity levels $A_{l,t}$. The consumer from island l consumes the goods produced in a subset of other islands. This subset is denoted by $\mathcal{B}_{l,t} \subset [0,1]$ and is randomly selected by nature each period. Symmetrically, the firms in island l are visited by a subset $\mathcal{C}_{l,t} \subset [0,1]$ of consumers coming from other islands. The random assignment of consumers to islands is such that the mass of goods in $\mathcal{B}_{l,t}$ is constant and so is the mass of consumers in $\mathcal{C}_{l,t}$. Labor is immobile across islands, so the consumer located in island l only works for the firms in island l.

Given this geography, I will make some crucial informational assumptions: agents in island l only observe productivity, output, prices and wages in their own island, the prices of the goods in the consumption basket of the local consumer, and a public noisy signal of aggregate inflation. With this information structure, agents only receive noisy price and quantity signals about the aggregate economy. The signal s_t, regarding the permanent component of the technology process is still present, and is publicly observed by all the agents in the economy.

Preferences and technology. Preferences are the same as in the previous section, except that consumption and labor supply now have an island index ($C_{l,t}$ and $N_{l,t}$) and the composite consumption good for the consumer from island l only includes the goods in the consumption basket $\mathcal{B}_{l,t}$, that is,

$$C_{l,t} = \left(\int_{\mathcal{B}_{l,t}} \int_0^1 C_{j,m,l,t}^{\frac{\gamma-1}{\gamma}} \, dj \, dm \right)^{\frac{\gamma}{\gamma-1}},$$

where $C_{j,m,l,t}$ is the consumption of variety j produced in island m, by the consumer from island l, at time t.

The production function is

$$Y_{j,l,t} = A_{l,t} N_{j,l,t}, \tag{18}$$

where $N_{j,l,t}$ is the labor input and $A_{l,t}$ is the island-specific productivity.

Uncertainty. As in the basic model of Section I, productivity $a_{l,t}$ has a permanent component and a temporary component, but the temporary component is now idiosyncratic to island l and is denoted by $\eta_{l,t}$. Therefore, $a_{l,t}$ is given by

$$a_{l,t} = x_t + \eta_{l,t}.$$

For each island l, the idiosyncratic shock, $\eta_{l,t}$, is normal, with zero mean and variance σ_η^2, serially uncorrelated, and independent of the aggregate shocks ϵ_t and e_t. The cross sectional distribution of $\eta_{l,t}$ satisfies $\int_0^1 \eta_{l,t} dl = 0$. The process for x_t and for the public signal s_t are given by (3) and (4), as in Section I.

Finally, there are two idiosyncratic shocks $\xi_{l,t}^1$ and $\xi_{l,t}^2$, which introduce noise in the endogenous price and quantity signals observed by the agents, and a shock ω_t to the public signal about aggregate inflation. For ease of exposition, I will discuss them in detail below.

Each period, consumers and firms located in island l choose quantities and prices optimally on the basis of the information available to them which includes: the local productivity $a_{l,t}$, the public signal s_t, the price of the one-period nominal bond Q_t, the local wage rate $W_{l,t}$, the prices of all the goods in the consumption basket of the local consumer $\{P_{j,m,t}\}_{j \in [0,1], m \in \mathcal{B}_{l,t}}$, the total sales of the local firms $\{Y_{j,l,t}\}_{j \in [0,1]}$, and the inflation index $\tilde{\pi}_t$, introduced below.

Consumers. The consumer in island l owns the firms in the island and, thus, receives the profits $\int_0^1 \Pi_{j,l,t} dj$, where $\Pi_{j,l,t}$ are the profits of firm j in island l. Nominal one-period bonds are the only financial assets traded across islands. Due to the presence of island-specific shocks, the consumer in island l is now subject to uninsurable idiosyncratic income shocks. His budget constraint is

$$Q_t B_{l,t+1} + \int_{\mathcal{B}_{l,t}} \int_0^1 P_{j,m,t} C_{j,m,l,t} \; dj \; dm = B_{l,t} + W_{l,t} N_{l,t} + \int_0^1 \Pi_{j,l,t} dj, \tag{19}$$

where $B_{l,t}$ denotes holdings of nominal bonds. In equilibrium, consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (19) and a no-Ponzi-game condition.

For each island, there are now two relevant price indexes. The first, is the local price index $P_{l,t}$, which includes all the goods produced in island l and is equal to

$$P_{l,t} = \left(\int_0^1 P_{j,l,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}.$$

The second, is the consumer price index $\overline{P}_{l,t}$, which includes all the goods consumed by consumer l, and is given by

$$\overline{P}_{l,t} = \left(\int_{\mathcal{B}_{l,t}} P_{m,t}^{1-\gamma} \; dm \right)^{\frac{1}{1-\gamma}}.$$

The demand for good j in island $m \in \mathcal{B}_{l,t}$ by consumer l is then

$$C_{j,m,l,t} = \left(\frac{P_{j,m,t}}{\overline{P}_{l,t}} \right)^{-\gamma} C_{l,t}.$$

Aggregating the demand of all consumers in $\mathcal{C}_{l,t}$, gives the demand for the good produced by firm j, l, which is equal to

$$Y_{j,l,t} = \int_{\mathcal{C}_{l,t}} \left(\frac{P_{j,l,t}}{\overline{P}_{m,t}} \right)^{-\gamma} C_{m,t} \; dm. \tag{20}$$

The economy-wide price index is defined, conventionally, as

$$P_t = \left(\int_0^1 P_{l,t}^{1-\gamma} \; dl \right)^{\frac{1}{1-\gamma}}.$$

Firms. Firms are price-setters *à la* Calvo (1983), as in the baseline model of Section

I. Each period, on each island, a fraction $1-\theta$ of firms are allowed to reset their price. Let $E_{l,t}[.]$ denote the expectation of the agents located in island l. Let $P^*_{l,t}$ denotes the optimal price for a firm who can adjust its price in island l at time t. The problem of this firm is to maximize

$$E_{l,t}\sum_{\tau=t}^{\infty}\theta^{\tau}Q^{l}_{t+\tau|t}\left(P_{j,l,t+\tau}Y_{j,l,t+\tau}-W_{l,t+\tau}N_{j,l,t+\tau}\right),$$

subject to $P_{j,l,t+\tau}=P^*_{l,t}$, the technological constraint (18) and the demand relation (20). The firm takes as given the stochastic processes for $W_{l,t}$ and for $\overline{P}_{m,t}$ and $C_{m,t}$ for all $m\in[0,1]$, and the stochastic discount factor of consumer l, given by $Q^{l}_{t+\tau|t}=\beta^{\tau}\left(C_{l,t+\tau}/C_{l,t}\right)^{-1}$.

Endogenous signals. Now I can discuss the endogenous price and quantity signals observed by the agents and introduce the sampling shocks $\xi^1_{l,t}$ and $\xi^2_{l,t}$. I assume that the random selection of islands in $\mathcal{B}_{l,t}$ is such that the consumer price index for island l is, in log-linear approximation,

$$\overline{p}_{l,t}=p_t+\xi^1_{l,t}, \tag{21}$$

where $\xi^1_{l,t}$ is i.i.d., normal, with zero mean and variance $\sigma^2_{\xi,1}>0$, and satisfies $\int_0^1\xi^1_{l,t}dl=0$. This assumption basically says that, each period, nature selects a biased sample of islands for each consumer l, so that the price index $\overline{p}_{l,t}$ is not identical to the aggregate price index p_t. The role of this assumption is to limit the ability of agents to infer the aggregate shocks from their observation of the prices of the goods they buy.[22]

The demand faced by firm j in island l, (20), can be rewritten, in log-linear approximation, as

$$y_{j,l,t}=\int_{m\in\mathcal{C}_{l,t}}\left(c_{m,t}+\gamma\overline{p}_{m,t}\right)dm-\gamma p_{j,l,t}.$$

I assume that the random selection of islands in $\mathcal{C}_{l,t}$ is such that the expression on the right-hand side takes the form

$$y_{j,l,t}=y_t-\gamma\left(p_{j,l,t}-p_t\right)+\xi^2_{l,t}, \tag{22}$$

where $\xi^2_{l,t}$ is i.i.d., normal, with zero mean and variance $\sigma^2_{\xi,2}>0$, and with $\int_0^1\xi^2_{l,t}dl=0$.

[22] Given that nature selects the island-prices $p_{\tilde{l},t}$ from the distribution $\{p_{\tilde{l},t}\}_{\tilde{l}\in[0,1]}$, consistency requires that the variance $\sigma^2_{\xi_1}$ is bounded above by the cross-sectional variance of prices across islands.

The underlying assumption is that the sample of consumers who buy goods in island l is a biased sample, so that firms only receive a noisy signal regarding $y_t + \gamma p_t$. Again, the role of this assumption is to limit the agents' ability to infer aggregate shocks by observing the quantities produced in their island.[23]

Monetary policy. Monetary policy is described by an interest rate rule which extends the rule in (8) by allowing for inertia in the response to inflation and for measurement error in the central bank's observation of current inflation. In particular, the nominal interest rate is given by

$$i_t = (1 - \rho_i)\, i^* + \rho_i i_{t-1} + \phi \tilde{\pi}_t, \tag{23}$$

where $\rho_i \in [0, 1)$ and ϕ are coefficients chosen by the monetary authority and $\tilde{\pi}_t$ is a noisy measure of realized aggregate inflation

$$\tilde{\pi}_t = \pi_t + \omega_t,$$

where ω_t is an i.i.d. normal shock, with zero mean and variance σ_ω^2. Consumers also observe $\tilde{\pi}_t$. In fact, given that they observe the nominal interest rate and there are no monetary policy shocks, they can perfectly infer $\tilde{\pi}_t$ from their observation of i_t and i_{t-1}.

An immediate interpretation of the shock ω_t is in terms of measurement error. The evidence in David E. Runkle (1998) and Dean Croushore (2007) shows that early releases of inflation data contain considerable amounts of noise.[24] However, the shock ω_t can also be interpreted more broadly as a stand-in for all those shocks and specification errors not explicitly modeled which make inflation a noisy measure of the distance between realized output and natural output (e.g., mark-up shocks). In Section B, I will discuss how the interpretation of the shock ω_t is relevant for the quantitative implications of the model.

In general, the role of the error terms $\xi_{l,t}^1, \xi_{l,t}^2$ and ω_t, is to limit the consumers' ability

[23] As in the case of prices, consistency requires that the variance $\sigma_{\xi_2}^2$ is bounded above by the cross-sectional variance of $c_{l,t} + \gamma \overline{p}_{l,t}$ across islands.

[24] Runkle (1998) looks at the implicit GDP deflator, Croushore (2007) at the personal consumption expenditure price index, which is the main inflation variable used by the Federal Reserve since 2000. Both evaluate measurement error comparing early releases of the price index with the revised series available at later dates. Measurement error in CPI inflation is harder to quantify, given that the CPI is not subject to revisions.

to infer the value of the underlying aggregate fundamentals from local and aggregate observations of prices and quantities. In a simple model, with a small number of aggregate shocks, it is relatively easy for consumers to back out the fundamentals from these observations. In practice, the consumers' ability to estimate the economy's fundamentals is likely to be impaired by the presence of a larger number of shocks, with richer patterns of serial correlation, by model misspecification, and by the possible presence of structural breaks. The approach of this paper is to keep the model relatively simple, assume that consumers know exactly the model's parameters, and to complicate their inference problem only by adding noise to their information.

B. *Equilibrium*

Unlike in the basic environment of Section I, the equilibrium dynamics of inflation and output can no longer be derived analytically, so the model will be solved numerically. As before, I study a log-linear approximation to a rational expectations equilibrium. In a setup with dispersed information a log-linear approximation helps in three dimensions: it simplifies the inference problem of the individual agents, it simplifies the state space for individual decision rules, and it simplifies aggregation.

Individual optimality conditions. Let me first derive the individual optimality conditions which will be used to characterize an equilibrium. The consumers' Euler equation takes the form[25]

$$c_{l,t} = E_{l,t}\left[c_{l,t+1}\right] - i_t + E_{l,t}\left[\overline{p}_{l,t+1}\right] - \overline{p}_{l,t}. \tag{24}$$

The two differences with equation (9) are that both expected consumption and expected inflation are island-specific. On the other hand, as I will argue in the next section, consumption in each island l still tends to converge towards the common level dictated by the permanent productivity x_t. Therefore, through the term $E_{l,t}\left[c_{l,t+1}\right]$, $c_{l,t}$ is still driven by the agents' expectations of x_t, as in Section I.

To complete the characterization of the consumption side, it is useful to write down

[25]To obtain log-linear approximations of the optimality conditions, I take as a reference point the stochastic equilibrium of an economy with no heterogeneity and full information (i.e., where all variances except σ_ϵ^2 are set to zero). The full derivations are in the online appendix.

the individual budget constraint in log-linearized form, which is

$$\beta h_{l,t+1} = h_{l,t} + p_{l,t} + y_{l,t} - \overline{p}_{l,t} - c_{l,t}, \tag{25}$$

where $h_{l,t} \equiv B_{l,t}/E_{l,t}\left[P_t Y_t\right]$ is the ratio of nominal bond holdings to expected aggregate nominal output. The variable $B_{l,t}$ is kept in levels rather than in logs, since it can take both positive and negative values.

Optimality for a firm who can update its price at date t gives

$$p^*_{l,t} = (1-\beta\theta) \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} E_{l,t}\left[w_{l,t+\tau} - a_{l,t+\tau}\right], \tag{26}$$

where $w_{l,t+\tau} - a_{l,t+\tau}$ represents the marginal cost in nominal terms in island l. This condition can be rewritten in recursive form as

$$p^*_{l,t} = (1-\beta\theta)\left(w_{l,t} - a_{l,t}\right) + \beta\theta E_{l,t}\left[p^*_{l,t+1}\right].$$

The law of motion for the local price index is

$$p_{l,t} = \theta p_{l,t-1} + (1-\theta)\, p^*_{l,t}.$$

Rearranging the last two equations and using the consumer's optimality condition for labor supply, $w_{l,t} - \overline{p}_{l,t} - c_{l,t} = \zeta n_{l,t}$, the labor market clearing condition, $n_{l,t} = y_{l,t} - a_{l,t}$, and the demand relation (22), I then obtain

$$p_{l,t} - p_{l,t-1} = \lambda\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \lambda\zeta\left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right) + \beta E_{l,t}\left[p_{l,t+1} - p_{l,t}\right], \tag{27}$$

where $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$ and

$$d_{l,t} \equiv y_t + \gamma p_t + \xi^2_{l,t}. \tag{28}$$

The quantity $d_{l,t}$ corresponds to the intercept of the demand function faced by the producers in island l in period t (in log-linear terms).

Expression (27) shows that prices in island l tend to increase when either the consumption of the local consumer or the demand for the goods produced in island l are high relative to the local productivity $a_{l,t}$. The consumption of the local consumer matters since it determines the location of the labor supply curve in island l. The demand of external consumers matters because it determines the amount of labor input required. Both variables jointly determine equilibrium wages and thus equilibrium marginal costs in island l.

The presence of imperfect information makes it impossible to aggregate (27) across islands and obtain a simple equation linking aggregate inflation to the aggregate output gap, as in (11). However, the underlying logic survives as I will show in Section III.

Learning and aggregation. The economy's aggregate dynamics will be described in terms of the variables $z_t = (x_t, e_t, p_t, i_t)$. The state of the economy is captured by the infinite dimensional vector $\mathbf{z}_t = (z_t, z_{t-1}, ...)$. I am looking for a linear equilibrium where the law of motion for $\mathbf{z}_t$ takes the form

$$\mathbf{z}_t = A\mathbf{z}_{t-1} + B\mathbf{u}_t^1, \tag{29}$$

with

$$\mathbf{u}_t^1 \equiv \begin{pmatrix} \epsilon_t & e_t & \omega_t \end{pmatrix}',$$

and the appropriate rows of A and B conform with the law of motion of x_t in (3) and with the monetary policy rule (23).[26]

To solve for a rational expectations equilibrium, I conjecture that $p_{l,t}$ and $c_{l,t}$ follow the rules

$$p_{l,t} = q_h h_{l,t} + q_p p_{l,t-1} + q_a a_{l,t} + q_d d_{l,t} + q_z E_{l,t}[\mathbf{z}_t], \tag{30}$$

$$c_{l,t} = -\overline{p}_{l,t} + b_h h_{l,t} + b_p p_{l,t-1} + b_a a_{l,t} + b_d d_{l,t} + b_z E_{l,t}[\mathbf{z}_t]. \tag{31}$$

The expressions (30) and (31) represent, respectively, the optimal pricing policy of the firms in island l (aggregated across firms) and the optimal consumption policy of the representative consumer in island l. Notice that $h_{l,t}$, $p_{l,t-1}$, and $E_{l,t}[\mathbf{z}_t]$ are the relevant individual state variables to describe the average behavior of consumers and firms in island l. I need to keep track of $h_{l,t}$ because of the consumer's budget constraint, I need $p_{l,t-1}$ because of Calvo pricing, and I need $E_{l,t}[\mathbf{z}_t]$ to form agents' expectations about

[26]See the appendix for explicit expressions for A and B.

current and future values of the aggregate state $\mathbf{z}_t$. The dynamics of $E_{l,t}[\mathbf{z}_t]$ can be characterized recursively using the Kalman filter,

$$E_{l,t}[\mathbf{z}_t] = AE_{l,t-1}[\mathbf{z}_{t-1}] + C(\mathbf{s}_{l,t} - E_{l,t-1}[\mathbf{s}_{l,t}]),$$

where $\mathbf{s}_{l,t}$ is the vector of signals observed by the agents in island l,[27]

$$\mathbf{s}_{l,t} = \begin{pmatrix} a_{l,t} & s_t & \overline{p}_{l,t} & d_{l,t} & i_t \end{pmatrix}',$$

and C is a matrix of Kalman gains, which is derived explicitly in the appendix. Let $\mathbf{z}_{t|t}$ denote the average expectation regarding the aggregate state $\mathbf{z}_t$, defined as

$$\mathbf{z}_{t|t} \equiv \int_0^1 E_{l,t}[\mathbf{z}_t]\, dl.$$

The individual updating rules can then be aggregated to find a matrix Ξ such that

$$\mathbf{z}_{t|t} = \Xi \mathbf{z}_t. \tag{32}$$

In equilibrium aggregate output y_t is given by

$$y_t = \psi \mathbf{z}_t, \tag{33}$$

where ψ is a vectors of constant coefficients. The solution of the model requires finding matrices A, B, C, Ξ, and vectors ψ, $\{q_h, q_p, q_a, q_d, q_z\}$, and $\{b_h, b_p, b_a, b_d, b_z\}$ that are consistent with agents' optimality, with Bayesian updating, and with market clearing in the goods, labor, and bonds markets. The details of the algorithm used for computations are in the appendix.

Computing an equilibrium requires dealing with the infinite histories $\mathbf{z}_t$. Here, I replace $\mathbf{z}_t$ with a truncated vector of states $\mathbf{z}_t^{[T]} = \{z_t, ..., z_{t-T}\}$. Numerical results show that when T is sufficiently large the choice of T does not affect the equilibrium dynamics. For the simulations presented below, I use $T = 50$. Kenneth Kasa (2000) uses frequency

[27] For computational reasons, it is convenient to include i_t instead of $\tilde{\pi}_t$ in the set variables observed by the agents. This allows me to simplify the Kalman filter, since, in this way, aggregate shocks do not appear in the observation equation. Since the agents know the monetary policy rule, (23), they can recover $\tilde{\pi}_t$ from their observation of i_t and i_{t-1}.

domain methods to deal explicitly with infinite histories and explores in what cases infinite histories lead to a fully revealing equilibrium. In particular, he shows that in the model of Townsend (1983) with a continuum of industries, imperfect information does not go away when looking at infinite histories.[28] My numerical results suggest that my model belongs to the same class of models, given that increasing T in my simulations does not lead to smaller expectational errors.[29]

III. Noise shocks and aggregate volatility

In this section, I use numerical simulations to explore the qualitative implications of the model with dispersed information. The model is too stylized for a full quantitative analysis, so I will only take a first pass at a basic quantitative question: how large are the cyclical movements generated by noise in public information? To address this question, I will compare the demand shocks generated by the model, under different parametrizations, with those generated by a simple bivariate VAR.

The discount factor β is set equal to 0.99, so the time period can be interpreted as a quarter. The inverse Frisch elasticity of labor supply ζ is set to 0.5 and the elasticity of substitution γ is set to 7.5. The parameter θ is set equal to 2/3, corresponding to an average price duration of three quarters. These values are in the range of those used in existing DSGE studies with monopolistic competition and sticky prices. The parameters for the monetary policy rule are set at $\rho_i = 0.9$ and $\phi = 1.5$, which corresponds to a relatively inertial Taylor rule with a response to inflation broadly consistent with existing empirical estimates.[30]

It remains to choose values for the variance parameters. Unlike standard linearized models, models with imperfect information do not display a "certainty equivalence" property, that is, variance parameters do not simply determine the size of the shocks, leaving the behavioral responses unchanged. These parameters also affect the filtering problem faced by the agents and thus contribute to determine the size and shape of the responses of the endogenous variables. I have chosen variance parameters so as to

[28] See Section 2 of Kasa (2000).

[29] See also Giacomo Rondina (2008) for an application of frequency domain methods to a related monetary environment and Nimark (2007), who uses a truncation method in the space of higher order expectations.

[30] See, for example, Richard Clarida et al. (2000), Christiano et al. (2005).

replicate as well as possible the impulse responses derived from a simple bivariate VAR in output and employment. The choice of these parameters is discussed in detail in B. The baseline parameters are reported in Table 1.

σ_ϵ	0.0077	σ_η	0.15
σ_e	0.03	$\sigma_{\xi,1}$	0.02
σ_ω	0.0006	$\sigma_{\xi,2}$	0.11

Table 1 – Baseline parameters

A. *Responses to the three shocks*

Figure 1 depicts the responses of output, employment, inflation and the interest rate to the three shocks ϵ_t, e_t and ω_t. In the first row of graphs, I plot both the response of y_t (solid line) and that of $y_{t+1|t} = \int E_{l,t}\left[y_{t+1}\right] dl$ representing the average expectation of next period's output y_{t+1} (dashed line). In the second row, I plot the response of hours. In the third row, I plot both the response of actual inflation π_t (solid line) and of the noisy inflation measure $\tilde{\pi}_t$ (dashed line). In the fourth row, I plot the nominal interest rate i_t (solid line) and the average expected real rate, given by $r_t = i_t - p_{t+1|t} + p_t$ (dashed line). For all three shocks, I plot the responses to a 1-standard-error shock.[31]

From a qualitative point of view, the shocks ϵ_t and e_t have similar effects as in the common information model of Section I. The technology shock leads to a gradual adjustment of output to its new long run level and to a temporary fall in employment and inflation. The noise shock e_t leads to a joint, temporary increase in output, employment and inflation.

The intuition behind the output response to the noise shock is that forward looking consumers expect their future incomes and consumption levels to be driven by the permanent, common component of technology, x_t. A noise shock temporarily increases their expectation of x_t. This increases expected future consumption on the right-hand side of the Euler equation (24).[32] At the same time, the real interest rate, also on the right-hand side of (24), responds sluggishly, due to the combination of nominal rigidi-

[31] In all figures, the scale of the responses is multiplied by 100 to make the graphs more readable.

[32] Aggregating across islands, this gives $\int E_{l,t}\left[c_{l,t+1}\right] dl$, which tends to move together with $y_{t+1|t}$. The two are not identical given that $y_{t+1|t} = \int E_{l,t}\left[\int c_{l,t+1} dl\right] dl$ and, under dispersed information, cross-sectional integration and the expectation operator are not interchangeable.

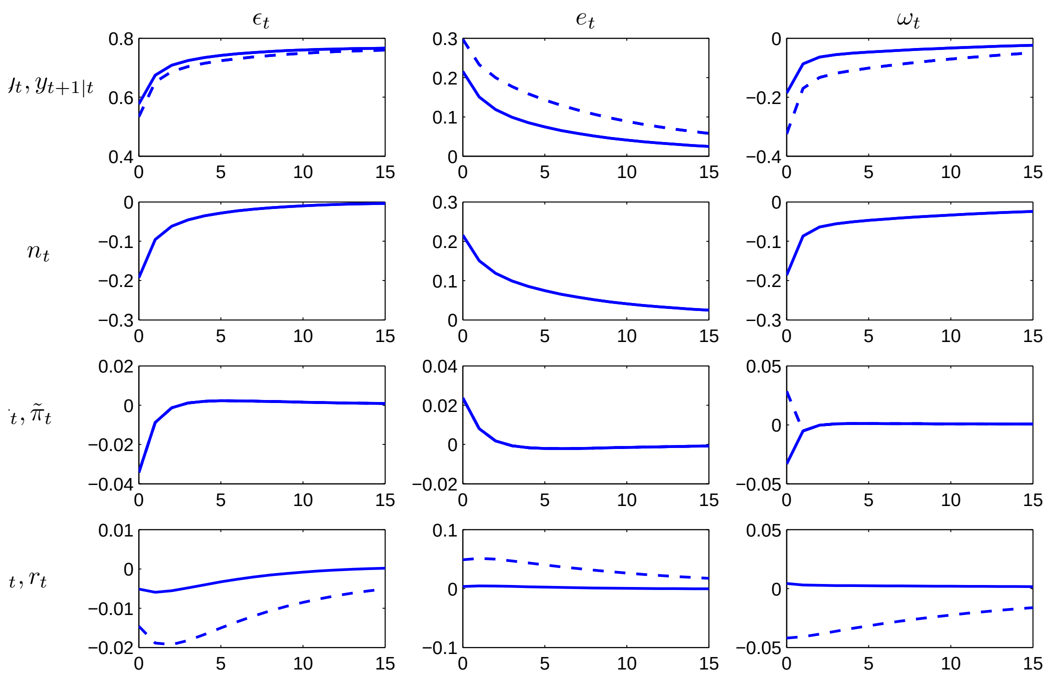

Figure 1. Impulse responses of output, employment, inflation and the interest rate.

ties and of a partially responsive monetary rule. This implies that the pressure from increased consumers' demand translates into a temporary increase in output and employment. The last row of graphs confirms this intuition, showing that the response of the real interest rate is relatively small after all three shocks, so that movements in consumption are dominated by movements in income expectations.

To understand the effects of the shock ω_t notice that this shock operates through two channels: a monetary policy channel and an information channel. First, a positive shock ω_t leads to a temporary increase in measured inflation and thus to a nominal interest rate increase. Second, agents know that positive inflation tends to be associated with an overestimate of natural output. Thus, the shock ω_t leads agents to revise downwards their expectations about x_t. Both channels lead to a reduction in spending and aggregate output. In particular, with the parameters given above, the information channel explains virtually all of the output decline following the shock. This can be seen both in the strong commovement of y_t and $y_{t+1|t}$ after an ω_t shock (in the top right graph of Figure 1) and in the small response of the nominal interest rate (in the bottom right graph). Therefore, in this example ω_t is essentially an additional noise shock, leading to qualitative responses analogous to those following an e_t shock (with the opposite sign).

This leads to an interesting observation. The presence of a noisy public statistic (here about inflation) can have ambiguous effects on noise-driven volatility. On the one hand, it allows agents to better estimate the economy's fundamentals. On the other hand, it introduces an additional source of correlated expectational errors. I will return to this point below and discuss further the role of the shock ω_t.

B. *How much noise-driven volatility can the model generate?*

In Figure 2, I report the impulse responses obtained from a simple bivariate VAR of GDP and hours, using US quarterly data. To identify the technology shock I use a long-run identification restriction *à la* Blanchard and Danny Quah (1989), following Galí (1999).[33] In Figure 3, I report the impulse responses obtained from performing

[33] The data are from the Haver USECON database, the sample period is 1948:1-2006:3, the measure of output is the GDP quantity index (GDPQ), and the measure of hours worked is hours in the business sector (LXBH). Both variables are in logs, multiplied by 100. Hours are detrended using a quadratic trend, and the VAR is estimated using 4 lags. The dashed lines represent 10% confidence bands, computed following Sims and Tao Zha (1999).

the same exercise on a 10,000 period sample generated using the simulated theoretical model.[34] The solid lines correspond to the baseline parameters in Table 1, the dashed lines to an alternative parametrization, with $(\sigma_e, \sigma_\omega) = (0.04, 0.0015)$ and the remaining parameters as in Table 1.

Comparing the top left panels of Figures 1 and 3 shows that the long-run identification strategy allows me to identify the productivity shock ϵ_t in the model, despite the presence of three shocks in the model. At the same time, the "non-technology" shock obtained from the VAR on the simulated series reflects the combined effect of the shocks e_t and ω_t. As argued above, both e_t and ω_t are essentially two noise shocks and have similar implications for output and employment. Therefore, here I will not attempt to separate them with a richer identification strategy and I will concentrate on evaluating their joint role in generating short-run volatility.

Notice also that, in comparing the model with the data, I am implicitly attributing all non-technology disturbances in the empirical VAR to noise shocks. This is clearly an extreme assumption, as it leaves out a number of additional shocks that one would like to include on the demand side (e.g., monetary shocks and shocks to government spending). However, this assumption is in line with the spirit of the exercise, which is to generate as much demand-side volatility as possible, using *only* noise shocks.

Let me discuss the choice of the variance parameters. The standard deviation of the technology shock, σ_ϵ, is set so as to replicate the empirical long-run output response to an identified technology shock in the data, which is 0.0077.

The noise shock volatility σ_e is chosen in order to obtain the maximum amount of noise-driven output volatility in the short run. That is, to maximize the impact response of output to non-technology shocks. Notice that there is a non-monotone relation between σ_e and noise-driven volatility, as in the common information model of Section I. As I increase the variance of the noise associated to the signal s_t, the precision of the signal deteriorates and agents put less weight on it. To illustrate this, in Figure 4, I plot the output response to the shocks ϵ_t and e_t, for different values of σ_e, keeping all other parameters at their baseline levels.[35] This figure illustrates the rich effects that

[34] See Fabio Canova (2007, Chapter 4.7) for a systematic discussion of this type of "first pass" evaluation of DSGE models using VARs.

[35] Recall that the impulse responses are normalized to represent responses to a 1-standard-deviation shock.

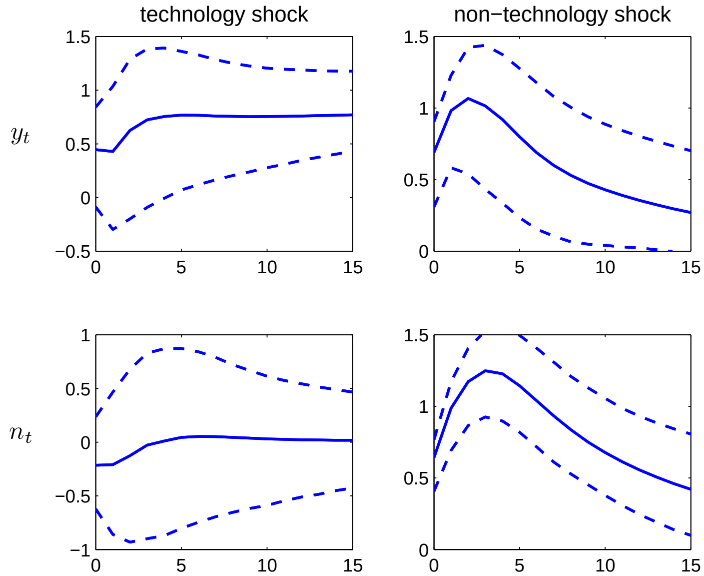

Figure 2. Impulse responses: bivariate VAR on US data, with a long-run restriction.

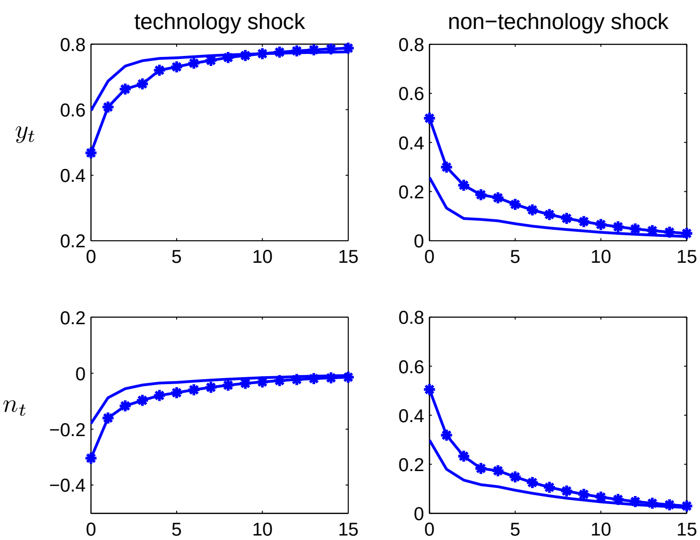

Figure 3. Impulse responses: bivariate VAR on simulated data, with a long-run restriction. Solid lines: baseline parametrization. Starred lines: high-variance parametrization.

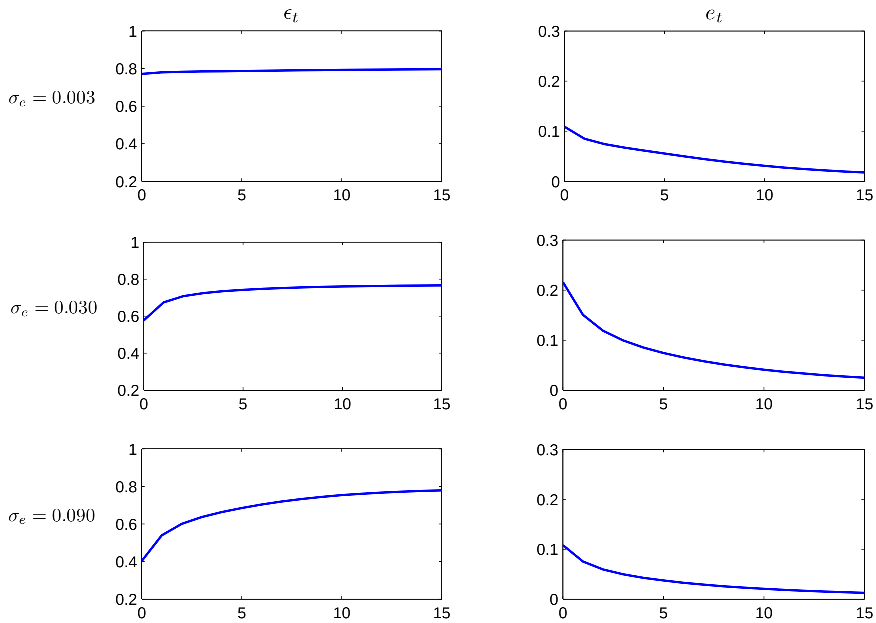

Figure 4. Output responses for different values of σ_e

the choice of variance parameters has on the model dynamics. The maximum output response to the e_t shock, arises for intermediate values of σ_e. Changing σ_e also modifies the shape of the output response to a productivity shock, by changing the speed at which agents learn the underlying value of x_t. When σ_e is larger, learning is slower and the output response to a technology shock is more gradual.

To choose σ_ω, I interpret ω_t as measurement error in early releases of inflation data and try to match the signal-to-noise ratio in these data. Namely, I interpret $\tilde{\pi}_t$ as the first release of the inflation rate and π_t as the final revision.[36] Then, I choose σ_ω to obtain a realistic value for the ratio between the standard deviation of the measurement error and the standard deviation of the innovation in "true" inflation. The latter is measured running a simple univariate autoregression of inflation with two lags (using data for the last revision). Runkle (1998) follows this approach and obtains a ratio of 0.77, using GDP deflator inflation. I performed similar calculations for PCE inflation, using the data in Croushore (2007), and obtained a ratio of 1.97.[37] Matching these two values, I obtain

[36]This is a conservative assumption, as the last revision will still contain some measurement error.

[37]Croushore (2007, Table 1) reports a standard deviation of the difference between the first release

the alternative parametrizations $(\sigma_e, \sigma_\omega) = (0.03, 0.0006)$ and $(\sigma_e, \sigma_\omega) = (0.04, 0.0015)$ depicted in Figure 3.

Finally, the idiosyncratic volatilities σ_η, $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$ are assumed to be large relative to the volatilities of the aggregate shocks. In particular, σ_η is taken to be 20 times as large as σ_ϵ. Given σ_η, $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$ are chosen so as to ensure that they satisfy the bounds described in footnotes 22 and 23.[38] As argued above, the assumption of large idiosyncratic shocks is not unrealistic, given recent empirical findings on firm-level volatility (see footnote 20). This choice of parameters implies that agents learn little about the underlying aggregate shocks from their local observation of productivity, prices and quantities and mostly learn from the exogenous signal s_t and from the endogenous signal $\tilde{\pi}_t$. As long as the values of $\sigma_\eta, \sigma_{\xi,1}$, and $\sigma_{\xi,2}$ are greater than or equal to those in Table 1, their specific values have little effect on the model's aggregate implications.

Comparing figures 2 and 3 shows that the model is able to capture well some features of the empirical impulse response functions. In particular, two patterns are present both in the data and in the model (under both parametrizations). First, following a non-technology shock, hours tend to increase roughly one-for-one with output. Second, following a technology shock, hours tend to fall and the drop in hours is of the same size as the difference between the impact response of output and its long-run response.[39]

There are other features of the empirical responses which are not captured by the model, in particular, the hump-shaped responses of output and hours following the non-technology shock. However, at this stage of the analysis, the major quantitative challenge for the model is whether it is able to generate relatively large non-technology disturbances. The answer to this question depends on the parametrization. In both

---

and the last revision of quarterly PCE inflation equal to 0.89. An autoregression of quarterly PCE inflation with two lags, gives a standard deviation of the innovation equal to 0.45. This gives me the ratio $0.98/0.45 = 1.97$. The difference with Runkle (1998) probably reflects the greater accuracy of PCE revisions.

[38] For a given value of σ_η, the parameters $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$ are chosen so as to ensure that they satisfy the bounds described in footnotes 22 and 23. In particular, I use (30) and (31) to evaluate the cross-sectional volatilities in prices and demand which are solely due to the idiosyncratic shocks $\eta_{l,t}$ and use these as conservative upper bounds for $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$.

[39] The finding that hours respond negatively to an identified technology shock is the subject of a heated debate (Christiano et al., 2003, Neville Francis and Valerie Ramey, 2003, V.V. Chari et al., 2004, Galí and Pau Rabanal, 2004). This debate has highlighted the need of a theory-based rationale for identification assumptions. The present model has at least the virtue of being consistent with the identification assumptions in the VAR exercise.

versions of the model the effects of non-technology shocks are smaller than in the data, but the model with larger volatilities for e_t and ω_t generates a larger amount of short-run volatility.[40] In particular, it generates an output response to the non-technology shock on impact which is not too far from the empirical one (0.0050 in the model, 0.0069 in the data). It also displays an output response to the technology shock more in line with the data, with a smaller effect on impact and a more gradual convergence afterwards. This is due to the fact that greater volatility in the public signals s_t and $\tilde{\pi}_t$ implies that the agents take more time to learn the value of x_t. Therefore, in terms of matching the empirical impulse responses, the second parametrization is preferable.

As argued above, the shock ω_t can be interpreted in several ways: as pure measurement error, or, more generally, as a "reduced-form" way of introducing shocks and specification errors which make inflation a noisy measure of the distance between current and natural output. Under this broader interpretation, one might parametrize the model with larger values of σ_ω, magnifying the effects of noise shocks.

In sum, the model ability to generate sizeable noise-driven shocks rests crucially on the assumptions made about the volatility of the various shocks. Clearly, the model is highly stylized so it is relatively easy for the agents in the model to figure out the underlying value of x_t from observing public statistics of productivity and inflation. A richer model which allows, for example, for monetary policy shocks and shocks to government expenditure, would make this inference problem more complicated. Monetary policy shocks would confound the inference problem of the agents, by breaking the tight connection between inflation and the estimation error of x_t. Therefore, introducing additional shocks might be useful not only to increase directly demand-side volatility, but also to magnify the effect of noise shocks.

So far I have concentrated on the model's ability to replicate *conditional* correlations of output and employment, which are estimated using VAR methods. An alternative is to evaluate the model looking at its implication in terms of simple *unconditional* correlations. It is interesting to recast the conclusions of this section in terms of unconditional implications. Notice that the model generates a negative correlation of output and employment following a technology shock and a positive correlation following a noise shock.

[40] This is not as obvious as it sounds, given the non-monotone relation between noise variance and noise driven volatility.

Therefore, to obtain a positive unconditional correlation (as observed in the data), I need noise shocks to explain a large fraction of total volatility. In particular, under the baseline parametrization described above I get a correlation between output and employment which is basically zero (looking at HP-filtered simulated series), while under the high-variance parametrization used for Figure 3, the correlation is 0.25. Both fall short of the empirical correlation which is around 0.8. The model does better in terms of total employment volatility. The standard deviation of (HP-filtered) hours relative to that of output is 0.37 in the baseline parametrization and 0.62 in the high-variance parametrization. The corresponding empirical value is close to 1. Therefore, both conditional and unconditional moments show that, although noise shocks can generate a sizeable fraction of demand-side volatility, they are not enough to explain all of it in the present model.

IV. A test using survey data

A crucial distinction between a noise shock and an actual technology shock is that, when a noise shock hits, agents' expectations tend to overreact, while, when a technology shock hits, they tend to underreact. In both cases agents receive a positive signal regarding x_t. However, given imperfect information, in the first case the actual change in x_t exceeds the expected change. In this case, firms lower prices, the expected real interest rate falls, and realized output ends up responding more than expected output. In the second case, firms tend to increase prices, the expected real interest rate increases, and realized output responds less. The difference $y_t - y_{t|t}$ is then positive in the first case and negative in the second case. This is illustrated in the top row of Figure 1.[41] This is a robust prediction of the model, which holds across a wide range of parameter values and captures the central role played by expectational errors in the model.

In this section, I attempt to test this prediction, using survey data to obtain a measure of the agents' average expectations, corresponding to $y_{t|t}$ in the model. The idea of the test is to use the simple bivariate semi-structural VAR introduced in the previous section to identify technology and non-technology shocks, and then to test the hypothesis that output expectations underreact following a technology shock and

[41] To help the Euler equation interpretation of consumption movements, the figure displays the path of the one-period-ahead average forecast $y_{t+1|t}$. However, the path for the contemporary average forecast $y_{t|t}$ is virtually identical to that of $y_{t+1|t}$.

overreact following a non-technology shock. As measures of expectations, I use data from the Survey of Professional Forecasters (SPF) and from the Michigan Consumer Sentiment Survey (CSS). From the SPF, I take the median forecasts of nominal GDP and of the GDP deflator in the coming quarter, to form a forecast of real GDP.[42] From the CSS, I take the third component of the Index of Consumer Sentiment which reflects the consumers' expectations regarding the state of the economy in the coming 12 months.[43]

To make the coefficients easier to interpret, I normalize the technology and the non-technology shock from the VAR, so that each has a 1% impact effect on output. Then, letting Y_t^e denote the expectation variable, I regress Y_t^e on contemporaneous values of the technology and non-technology shocks, on lagged values of Y_t^e, and on lagged values of GDP and hours.[44] In Table 3, I report the regression coefficients of the two shocks, for each data set. In the same table, I also report the F-statistic for a test of the difference between the coefficients of the two shocks.

Survey of Professional Forecasters Data		
	Coefficient	Standard error
Technology shock	0.9063	0.2515
Non-technology shock	1.5582	0.1582

Test of the difference between coefficients
F(1,133) = 4.7144 (significance level: 0.0317)

Consumer Sentiment Survey Data		
	Coefficient	Standard error
Technology shock	404.46	219.78
Non-technology shock	644.76	149.18

Test of the difference between coefficients
F(1,167) = 0.8692 (significance level: 0.3525)

Table 1. Regression of expectation measures on identified shocks

These results lend support to the model's prediction, as both the coefficients for the technology and the non-technology shock are positive and the coefficient for the non-

[42]The mnemonics for the two variables used are NGDP2 and PGDP2. The sample is 1968:4-2006:3.
[43]This is the component denominated "Business Condition, 12 months." The sample is 1960:1-2006:3.
[44]In the reported regressions I use 4 lags for Y_t^e, GDP and hours.

technology shock is larger using both data sets. However, the difference between the two coefficients is only significant when I use the SPF data. This may be due to the fact that the SPF variable is defined in terms of an explicit estimate of aggregate GDP, while the CSS index is an aggregate of qualitative responses, which is more loosely connected to the respondents' quantitative expectations about aggregate output.[45] This also implies that the values of the coefficients have a more meaningful interpretation in the case of the SPF data. In this case, the coefficients can be used for a stronger test of the model's predictions. That is, I can look at the absolute value of the two coefficients and not just at their difference. Also this version of the test provides support to the model's mechanism. In particular, the coefficient on the technology shock is smaller than 1 (although not significantly different from 1), and the coefficient on the non-technology shock is larger than 1 (and here the difference is statistically significant).

To check the robustness of the result, I have repeated the exercise using different VAR specifications to estimate the technology and non-technology shocks, obtaining very similar results.[46] I have also tried different measures of expectations, obtaining similar qualitative results, although the difference in the coefficients is only significant when using one and two-quarter-ahead SPF forecasts.[47]

Notice that the VAR identification approach used here bunches together all non-technology shocks as if they were all driven by noise. Richer identification strategies, which are able to tease out monetary shocks and shocks to government spending, may yield sharper conclusions regarding the effect of noise shocks on expectations.

V. Concluding remarks

In this paper, I interpret the business cycle as a process of noisy learning by the consumers, who can temporarily overstate or understate the economy's productive capacity.

[45] The index used here is based on the respondents answers to the question: "Now turning to business conditions in the country as a whole–do you think that during the next twelve months we'll have good times financially, or bad times, or what?" The index is equal to the percentage of positive answers minus the percentage of negative answers (plus 100).

[46] I have tried a VAR using total employment (LE) and hours in the non-farm sector (LXNFH), and I have used output in the business and non-farm sectors instead of GDP (LXBO, LXNFO) paired with the respective hours' measure. I have also tried a bivariate VAR of output and inflation, still using a long-run restriction to identify the technology shock.

[47] I have tried the SPF median forecasts of output in the coming 2 to 3 quarters and I have tried the index of "Business Conditions, 5 years," from the Consumer Sentiment Survey.

This idea is incorporated into a standard dynamic general equilibrium model and gives rise to noise shocks which have the features of aggregate demand shocks.

For the sake of simplicity, the model makes sharp assumptions on the processes for aggregate and individual technology: the aggregate technology shock is a permanent level shock, the idiosyncratic shock is purely a temporary shock. Both assumptions could be relaxed. Introducing persistent shocks to the growth rate of aggregate TFP, rather than to the level, may help to better capture the uncertainty about medium run swings in productivity growth. This modification may lead to potentially larger consumption responses, given that the same short run increase in TFP would be associated to larger increases in the expected present value of income. Introducing persistence in idiosyncratic shocks would have two effects. On the one hand, it would induce agents to rely more on their private productivity signal, since it is a better predictor of future individual income. This would tend to reduce the effect of noise shocks. On the other hand, it would introduce serial correlation in the private productivity signal, inducing slower learning and increasing the effect of noise shocks.

Also, the model features no capital and has a very limited role for financial markets. This choice was made to concentrate on the consumers' learning dynamics and to introduce dispersed information in the simplest setup. However, uncertainty about long-run technology innovations is clearly crucial for investment decisions. Adding investment may help to generate larger demand responses following a noise shocks, improving the model's ability to fit the data.

Finally, the model requires high levels of idiosyncratic uncertainty in order to generate relatively slow aggregate learning. One reason for this is that the agents in the model know exactly the model's structure and have unlimited capacity to acquire and process information. It would be interesting to extend the model to relax these assumptions. In particular, the model is well suited to the introduction of limited attention *à la* Sims (2003).[48]

The analysis in Section III suggests that noise shocks may generate sizeable levels of short run volatility. However, a number of issues remain open. In particular, how should one calibrate the idiosyncratic noise in the private signals observed by individual agents and the aggregate noise in the public signals? Is it possible to obtain direct measures of

[48]See Maćkowiak and Wiederholt (2007).

noise shocks to test their effect directly?

To calibrate idiosyncratic noise, one possibility is to look at measures of cross-sectional dispersion in quantities and prices. However, this is subject to the caveat that agents' private information is probably richer than just individual price and quantity observations. Moreover, limited attention may imply that not all the private information is efficiently processed to forecast aggregate changes in fundamentals. An alternative approach may be to calibrate idiosyncratic noise by looking at measures of cross-sectional dispersion in expectations, obtained from survey data.

To calibrate the noise in aggregate signals, an approach is to use data on revision errors in early releases of aggregate statistics, as I did in section B for the inflation signal. These can also be used for direct tests of the transmission of noise shocks, as in the papers by Oh and Waldman (1990) and Rodriguez Mora and Schulstad (2007) mentioned in the introduction. However, aggregate statistics are only a subset of the public signals available to the private sector. Again, survey-based data on average expectations may help calibrate the total volatility in common noise. They can also be used for direct testing of the model implications, which is the strategy I followed in Section IV.

Finally, an alternative approach is to estimate the model solely looking at its implications for aggregate macroeconomic variables, as it is commonly done in estimated DSGE exercises. In this paper, it was possible to use a simple long-run identification assumption to compare the model with time series evidence on output and employment. In richer models, this is less likely to be feasible and interesting questions open up about estimation and identification in environments where agents have imperfect information on aggregate shocks.[49]

[49] See Collard et al. (2007) and Blanchard et al. (2009).

VI. Appendix

In this appendix, I provide a more detailed characterization of the equilibrium of the model with dispersed information and describe the algorithm used for computations. The matrices A and B are given by

$$A = \begin{bmatrix} 1 \quad \mathbf{0} \\ 0 \quad \mathbf{0} \\ A_p \\ \begin{pmatrix} 0 & 0 & -\phi & \rho_i & \mathbf{0} \end{pmatrix} + \phi A_p \\ I \quad \mathbf{0} \end{bmatrix}, \quad B = \begin{bmatrix} 1 \quad 0 \quad 0 \\ 0 \quad 1 \quad 0 \\ B_p \\ \phi \left(B_p + \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) \\ \mathbf{0} \end{bmatrix},$$

where A_p and B_p are vectors to be determined.

A. *Optimal decision rules*

First, let me substitute the demand relation (22) (aggregated across the firms in island l) to rewrite the budget constraint (25) as

$$\beta h_{l,t+1} = h_{l,t} + (1 - \gamma) p_{l,t} + d_{l,t} - \overline{p}_{l,t} - c_{l,t}, \tag{34}$$

where $d_{l,t}$ is defined by (28).

Prices. Using (30) to substitute for $p_{l,t+1}$ on the right-hand side of the optimal pricing condition (27) gives

$$\begin{aligned} \Lambda p_{l,t} &= p_{l,t-1} - \lambda (1 + \zeta) a_{l,t} + \lambda \left(\overline{p}_{l,t} + c_{l,t} \right) + \lambda \zeta d_{l,t} + \\ &\quad + \beta \left(q_h h_{l,t+1} + q_p p_{l,t} + q_a E_{l,t} \left[a_{l,t+1} \right] + q_d E_{l,t} \left[d_{l,t+1} \right] + q_z E_{l,t} \left[\mathbf{z}_{t+1} \right] \right) \end{aligned}$$

where $\Lambda \equiv 1 + \beta + \lambda (1 + \gamma \zeta)$. Use the budget constraint (34) to substitute for $h_{l,t+1}$ and (28) to substitute for $d_{l,t+1}$. The expected values of all aggregate variables dated $t + 1$ on the right-hand side can be expressed in terms of $\mathbf{z}_t$, using (29), (33), and the fact that $E_{l,t} \left[\mathbf{u}_{t+1}^1 \right] = 0$. Moreover, the expected values of all idiosyncratic shocks dated

$t+1$ are zero. Rearranging, I then obtain

$$\begin{aligned}\left(\Lambda-(1-\gamma)\,q_h-\beta q_p\right)p_{l,t} &= p_{l,t-1}+q_h h_{l,t}-\lambda\,(1+\zeta)\,a_{l,t}+\\ &+(\lambda\zeta+q_h)\,d_{l,t}+(\lambda-q_h)\left(\overline{p}_{l,t}+c_{l,t}\right)+\\ &+\beta\left(q_a\mathbf{e}_x+q_d\left(\psi+\gamma\mathbf{e}_p\right)+q_z\right)AE_{l,t}\left[\mathbf{z}_t\right].\end{aligned}$$

where $\mathbf{e}_x$ and $\mathbf{e}_p$ are unitary vectors which select, respectively, x_t and p_t from the state vector $\mathbf{z}_t$. Use (30) to substitute for $p_{l,t}$ on the left-hand side and (31) to substitute for $c_{l,t}$ on the right-hand side. Matching coefficients, this gives

$$q_p = \frac{1}{\Lambda-(1-\gamma)\,q_h-\beta q_p}, \tag{35a}$$

$$q_h = q_p\left(q_h+(\lambda-q_h)\,b_h\right), \tag{35b}$$

$$q_a = q_p\left(-\lambda\,(1+\zeta)+(\lambda-q_h)\,b_a\right), \tag{35c}$$

$$q_d = q_p\left(\lambda\zeta+q_h+(\lambda-q_h)\,b_d\right), \tag{35d}$$

$$q_z = q_p\left[\beta\left(q_a\mathbf{e}_x+q_d\left(\psi+\gamma\mathbf{e}_p\right)+q_z\right)A+(\lambda-q_h)\,b_z\right]. \tag{35e}$$

Consumption. Using (31) to substitute for $c_{l,t+1}$ on the right-hand side of the Euler equation (24) gives

$$c_{l,t}=-\overline{p}_{l,t}-i_t+b_h h_{l,t+1}+b_p p_{l,t}+b_a E_{l,t}\left[a_{l,t+1}\right]+b_d E_{l,t}\left[d_{l,t+1}\right]+b_z E_{l,t}\left[\mathbf{z}_{t+1}\right].$$

As in the case of prices, use the budget constraint (34) to substitute for $h_{l,t+1}$ and (28) to substitute for $d_{l,t+1}$. The expected values of future aggregate variables can be expressed in terms of $\mathbf{z}_t$, using (29), (33), and the fact that $E_{l,t}\left[\mathbf{u}^1_{t+1}\right]=0$. The expected values of future idiosyncratic shocks are zero. The resulting expression is

$$\begin{aligned}c_{l,t} &= -\overline{p}_{l,t}-i_t+\frac{b_h}{\beta}\left(h_{l,t}+d_{l,t}+(1-\gamma)\,p_{l,t}-\overline{p}_{l,t}-c_{l,t}\right)+\\ &+b_p p_{l,t}+\left(b_a\mathbf{e}_x+b_d\left(\psi+\gamma\mathbf{e}_p\right)+b_z\right)AE_{l,t}\left[\mathbf{z}_t\right],\end{aligned}$$

where $\mathbf{e}_x$ and $\mathbf{e}_p$ are the unitary vectors defined above. Rearranging, gives

$$\begin{aligned} c_{l,t} &= -\overline{p}_{l,t} + \frac{b_h}{\beta + b_h} h_{l,t} + \frac{b_h (1-\gamma) + \beta b_p}{\beta + b_h} p_{l,t} + \frac{b_h}{\beta + b_h} d_{l,t} + \\ &\quad + \frac{\beta}{\beta + b_h} \left[-\mathbf{e}_i + \left(b_a \mathbf{e}_x + b_d \left(\psi + \gamma \mathbf{e}_p \right) + b_z \right) A \right] E_{l,t} \left[\mathbf{z}_t \right], \end{aligned}$$

where $\mathbf{e}_i$ is the unitary vectors which selects i_t from $\mathbf{z}_t$. Use (31) to substitute for $c_{l,t}$ on the left-hand side and (30) to substitute for $p_{l,t}$ on the right-hand side. Matching coefficients, this gives

$$b_h = \frac{b_h}{\beta + b_h} + \varkappa q_h, \tag{36a}$$

$$b_p = \varkappa q_p, \tag{36b}$$

$$b_a = \varkappa q_a, \tag{36c}$$

$$b_d = \frac{b_h}{\beta + b_h} + \varkappa q_d, \tag{36d}$$

$$b_z = \frac{\beta}{\beta + b_h} \left[-\mathbf{e}_i + \left(b_a \mathbf{e}_x + b_d \left(\psi + \gamma \mathbf{e}_p \right) + b_z \right) A \right] + \varkappa q_z, \tag{36e}$$

where

$$\varkappa \equiv \frac{b_h (1-\gamma) + \beta b_p}{\beta + b_h}.$$

B. *Individual inference*

To find the Kalman gains for the individual learning problem notice that the vector of signals $\mathbf{s}_{l,t} = \begin{pmatrix} a_{l,t} & s_t & \overline{p}_{l,t} & d_{l,t} & i_t \end{pmatrix}'$ can be written as

$$\mathbf{s}_{l,t} = F \mathbf{z}_t + G \mathbf{u}_{l,t}^2$$

where

$$\mathbf{u}_{l,t}^2 \equiv \begin{pmatrix} \eta_{l,t} & \xi_{l,t}^1 & \xi_{l,t}^2 \end{pmatrix}',$$

the matrices F and G are

$$F \equiv \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_x + \mathbf{e}_e \\ \mathbf{e}_p \\ \psi + \gamma \mathbf{e}_p \\ \mathbf{e}_i \end{bmatrix}, G \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$\mathbf{e}_x, \mathbf{e}_p$, and $\mathbf{e}_i$ where defined above, and $\mathbf{e}_e$ is the unitary vector which selects e_t from $\mathbf{z}_t$. Bayesian updating for island l's agents implies that

$$E_{l,t}[\mathbf{z}_t] = E_{l,t-1}[\mathbf{z}_t] + C(\mathbf{s}_{l,t} - E_{l,t-1}[\mathbf{s}_{l,t}]).$$

Define the variance-covariance matrices

$$\Sigma = \begin{bmatrix} \sigma_\epsilon^2 & 0 & 0 \\ 0 & \sigma_\nu^2 & 0 \\ 0 & 0 & \sigma_\omega^2 \end{bmatrix}, V = \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_{\xi,1}^2 & 0 \\ 0 & 0 & \sigma_{\xi,2}^2 \end{bmatrix},$$

and let Ω be defined as

$$\Omega = Var_{l,t-1}[\mathbf{z}_t].$$

Then the Kalman gains C are given by

$$C = \Omega F' (F \Omega F' + GVG')^{-1}, \tag{37}$$

and Ω must satisfy the Riccati equation

$$\Omega = A(\Omega - CF\Omega)A' + B\Sigma B'. \tag{38}$$

C. *Fixed point*

The average first order expectations regarding the state $\mathbf{z}_t$ can be expressed as a function of $\mathbf{z}_t$ itself as

$$\mathbf{z}_{t|t} = \Xi \mathbf{z}_t.$$

Using the updating equations and aggregating across consumers gives:

$$\mathbf{z}_{t|t} = (I - CF)\, A\mathbf{z}_{t-1|t-1} + CF\mathbf{z}_t.$$

Therefore, the matrix Ξ must satisfy the condition

$$\Xi\mathbf{z}_t = (I - CF)\, A\Xi\mathbf{z}_{t-1} + CF\mathbf{z}_t, \tag{39}$$

for all $\mathbf{z}_t$. Aggregating the individual decision rules (30) and (31), I then obtain

$$\begin{aligned} p_t &= q_p p_{t-1} + q_a x_t + q_d\,(c_t + \gamma p_t) + q_z \Xi \mathbf{z}_t, \\ c_t &= -p_t + b_p p_{t-1} + b_a x_t + b_d\,(c_t + \gamma p_t) + b_z \Xi \mathbf{z}_t. \end{aligned}$$

Expressing everything in terms of the state $\mathbf{z}_t$, the equilibrium coefficients must satisfy

$$\left[\mathbf{e}_p - q_p \mathbf{e}_{p_{-1}} - q_a \mathbf{e}_x - q_d\,(\psi + \gamma \mathbf{e}_p) - q_z \Xi\right] \mathbf{z}_t = 0, \tag{40}$$

$$\left[\mathbf{e}_p + \psi - b_p \mathbf{e}_{p_{-1}} - b_a \mathbf{e}_x - b_d\,(\psi + \gamma \mathbf{e}_p) - b_z \Xi\right] \mathbf{z}_t = 0, \tag{41}$$

for all $\mathbf{z}_t$.

An equilibrium is characterized by the vectors A_p, B_p, ψ describing the aggregate dynamics, the vectors $\{q_h, q_p, q_a, q_d, q_z\}$, and $\{b_h, b_p, b_a, b_d, b_z\}$ describing individual behavior, the matrices C and Ω describing the individual learning problem, and the matrix Ξ capturing the aggregate behavior of first order expectations. These objects characterize an equilibrium if they satisfy conditions (35)-(41).

D. *Computation*

To compute an equilibrium I apply the following algorithm. I start with some initial value for $\{A_p, B_p, \psi, \}$. I derive the values of $\{q_h, q_p, q_a, q_d, q_z\}$ and $\{b_h, b_p, b_a, b_d, b_z\}$ which satisfy individual optimality, by substituting the prices and quantities (30) and (31) into (27), (24), and (25). Next, I solve for C and Ω in the individual inference problem. Since the vector $\mathbf{z}_t^{[T]}$ is truncated, I set to zero the value of z_{t-T-1} in $\mathbf{z}_{t-1}^{[T]}$ and replace (39) with

$$\Xi\mathbf{z}_t^{[T]} = (I - CF)\, A\Xi M\mathbf{z}_t^{[T]} + CF\mathbf{z}_t^{[T]},$$

where

$$M \equiv \begin{bmatrix} \mathbf{0} & I \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

This gives the following relation, which is used iteratively to compute Ξ,

$$\Xi = (I - CF)\, A \Xi M + CF.$$

I then apply the updating rule

$$\begin{aligned} A_p &= \left(q_p \mathbf{e}_{p-1} + q_a \mathbf{e}_x + q_d \left(\psi + \gamma \mathbf{e}_p\right) + q_z \Xi\right) A, \\ B_p &= \left(q_p \mathbf{e}_{p-1} + q_a \mathbf{e}_x + q_d \left(\psi + \gamma \mathbf{e}_p\right) + q_z \Xi\right) B, \\ \psi &= b_p \mathbf{e}_{p-1} + b_a \mathbf{e}_x + b_d \left(\psi + \gamma \mathbf{e}_p\right) + b_z \Xi - \mathbf{e}_p, \end{aligned}$$

and repeat until convergence is achieved. The convergence criterion is given by the quadratic distance of the impulse response functions of y_t and p_t to the shocks in $\mathbf{u}_t^1$ (with weights given by the variances of the shocks), under the old and updated values of $\{A_p, B_p, \psi\}$.

References

[1] Adam, Klaus. 2007. "Optimal Monetary Policy with Imperfect Common Knowledge." *Journal of Monetary Economics*, 54(2): 267-301.

[2] Allen, Franklin, Stephen Morris and Hyun Song Shin. 2006. "Beauty Contests, Bubbles and Iterated Expectations in Asset Markets." *Review of Financial Studies,* 19: 719-752.

[3] Amato, Jeffery, Stephen Morris and Hyun Song Shin, 2002, "Communication and Monetary Policy." *Oxford Review of Economic Policy* 18, 495-503.

[4] Angeletos, George-Marios and Alessandro Pavan. 2004. "Transparency of Information and Coordination in Economies with Investment Complementarities." *American Economic Review* 94(2).

[5] Aoki, Kosuke. 2003. "On the Optimal Monetary Policy Response to Noisy Indicators." *Journal of Monetary Economics,* 50: 501-523.

[6] Bacchetta, Philippe and Eric Van Wincoop. 2006. "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review* 96: 552-576.

[7] Barsky, Robert B. and Eric R. Sims. 2008. "Information Shocks, Animal Spirits, and the Meaning of Innovations in Consumer Confidence." mimeo, University of Michigan.

[8] Beaudry, Paul and Franck Portier. 2004. "Exploring Pigou's Theory of Cycles." *Journal of Monetary Economics*, 51(6): 1183-1216.

[9] Beaudry, Paul and Franck Portier. 2006. "Stock Prices, News and Economic Fluctutations." *American Economic Review*, 96(4), 1293-1307.

[10] Blanchard, Olivier J. 1993. "Consumption and the Recession of 1990-1991." *American Economic Review, Papers and Proceedings*, 83(2): 270-275.

[11] Blanchard, Olivier J., and Danny Quah. 1989. "The Dynamic Effects of Aggregate Demand and Aggregate Supply Shocks." *American Economic Review* 79(4): 655-673.

[12] Blanchard, Olivier J., Jean-Paul L'Huillier, and Guido Lorenzoni, 2009, "News, Noise, and Fluctuations: An Empirical Exploration." mimeo, MIT.

[13] Bomfim, Antulio N. 2001. "Measurement Error in General Equilibrium: The Aggregate Effects of Noisy Economic Indicators." *Journal of Monetary Economics*, 48(3): 585-603.

[14] Calvo, Guillermo. 1983. "Staggered Prices and in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3): 383-98.

[15] Canova, Fabio. 2007. *Methods for Applied Macroeconomic Research*, Princeton: Princeton University Press.

[16] Chari, V. V., Patrick J. Kehoe and Ellen R. McGrattan. 2004. "Are Structural VARs Useful Guides for Developing Business Cycle Theories?" Federal Reserve Bank of Minneapolis, Working Paper 631.

[17] Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy', *Journal of Political Economy*, 113(1): 1-45.

[18] Christiano, Lawrence J., Martin Eichenbaum, and Robert Vigfusson. 2003. "What happens after a Technology Shock?" NBER Working Paper 9819.

[19] Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2006. "Monetary Policy and Stock Market Boom-Bust Cycles." mimeo, Northwestern University.

[20] Clarida, Richard, Jordi Galí, and Mark Gertler. 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*, 115: 147-180.

[21] Cochrane, John H. 1994. "Shocks." *Carnegie-Rochester Conference Series on Public Policy*, 41: 295-364.

[22] Coibion, Olivier and Yuriy Gorodnichenko. 2008. “What Can Survey Forecasts Tell Us About Informational Rigidities?” mimeo, UC Berkeley.

[23] Collard, Fabrice and Harris Dellas. 2004. “The New Keynesian Model with Imperfect Information and Learning.” IDEI-Toulouse Working Paper 273.

[24] Collard, Fabrice, Harris Dellas, and Frank Smets. 2007. “Imperfect Information and the Business Cycle.” mimeo, University of Berne.

[25] Comin, Diego and Thomas Philippon. 2005. “The Rise in Firm-Level Volatility: Causes and Consequences.” *NBER Macroeconomics Annual.*

[26] Croushore, Dean. 2007. “Revisions to PCE Inflation Measures: Implications for Monetary Policy.” mimeo, University of Richmond.

[27] Danthine, Jean-Pierre, John Donaldson, and Thore Johnsen. 1998. “Productivity Growth, Consumer Confidence, and the Business Cycle.” *European Economic Review*, 42: 1113-1140.

[28] Edge, Rochelle M., Thomas Laubach, and John C. Williams. 2007. “Learning and Shifts in Long-run Productivity Growth.” *Journal of Monetary Economics*, 54(8): 2421-38.

[29] Franco, Francesco and Thomas Philippon, 2007, “Firms and Aggregate Dynamics.” *Review of Economics and Statistics*, 89(4): 587–600.

[30] Francis, Neville and Valerie Ramey. 2003. “Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited.” NBER Working Paper No. 8726.

[31] Galí, Jordi. 1999. “Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” *American Economic Review*: 249-271.

[32] Gilchrist, Simon and John V. Leahy. 2002. “Monetary Policy and Asset Prices.” *Journal of Monetary Economics,* 49 (1), 75-97.

[33] Hellwig, Christian. 2002. "Public Announcements, Adjustments Delays and the Business Cycle." mimeo UCLA.

[34] Hellwig, Christian. 2005. "Heterogeneous Information and the Benefits of Public Information Disclosures." mimeo UCLA.

[35] Jaimovich, Nir and Sergio Rebelo. 2006. "Can News about the Future Drive the Business Cycle?" mimeo, Northwestern University.

[36] Kasa, Kenneth. 2000. "Forecasting the Forecasts of Others in the Frequency Domain." *Review of Economic Dynamics,* 3: 726-56.

[37] Kawamoto, Takuji. 2004. "Technology Shocks and the Role of Monetary Policy in the Beauty Contest Monetarist Model." Bank of Japan Discussion Paper, 2004-E-10.

[38] Kydland, Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50 (6), 1345-1370.

[39] Lorenzoni, Guido, 2010, "Optimal Monetary Policy with Uncertain Fundamentals and Dispersed Information." *Review of Economic Studies.*

[40] Lucas, Robert E. Jr. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4(2): 103-24.

[41] Luo, Yulei. 2006. "Consumption Dynamics under Information Processing Constraints." *Review of Economic Dynamics*, 11(2): 366-385.

[42] Maćkowiak, Bartosz and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review*, forthcoming.

[43] Maćkowiak, Bartosz and Mirko Wiederholt. 2007. "Macroeconomic Dynamics under Rational Inattention." mimeo, Northwestern University.

[44] Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117: 1295-1328.

[45] Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, 2003, "Disagreement about Inflation Expectations." *NBER Macroeconomics Annual*, 18: 209-248.

[46] Milani, Fabio. 2007. "Expectations, Learning and Macroeconomic Persistence." *Journal of Monetary Economics,* 54(7): 2065-82.

[47] Morris, Stephen and Hyun Song Shin. 2002. "The Social Value of Public Information." *American Economic Review*, 92: 1521-1534.

[48] Moscarini, Giuseppe. 2004. "Limited Information Capacity as a Source of Inertia." *Journal of Economic Dynamics and Control*, 28(10): 2003-2035.

[49] Muth, John F. 1960. "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association*

[50] Nimark, Kristoffer P. 2007. " Dynamic Pricing and Imperfect Common Knowledge." *Journal of Monetary Economics*, 55: 365-382.

[51] Nimark, Kristoffer P. 2008. " Dynamic Higher Order Expectations." mimeo, Reserve Bank of Australia.

[52] Oh, Seonghwan and Michael Waldman. 1990. "The Macroeconomic Effects of False Announcements." *Quarterly Journal of Economics*, 105 (4): 1017-34.

[53] Orphanides, Athanasios. 2003. "Monetary Policy Evaluation with Noisy Information." *Journal of Monetary Economics*, 50(3): 605-631.

[54] Phelps, Edmund S. et al. 1969. *Microeconomic Foundations of Employment and Inflation Theory*. New York: Norton.

[55] Pigou, Arthur C., 1926. *Industrial Fluctuations.* MacMillan, London.

[56] Primiceri, Giorgio, Ernst Schaumburg and Andrea Tambalotti. 2006. "Intertemporal Disturbances." NBER Working Paper No. 12243.

[57] Reis, Ricardo. 2003. "Where Is the Natural Rate? Rational Policy Mistakes and Persistent Deviations of Inflation from Target." *Advances in Macroeconomics, B. E. Journals in Macroeconomics*, 3(1).

[58] Reis, Ricardo. 2006. "Inattentive Producers." *Review of Economic Studies*, 73(3).

[59] Rodriguez Mora, José V. and Paul Schulstad. 2007. "The Effect of GNP Announcements on Fluctuations of GNP Growth." *European Economic Review*, 51(8): 1922-1940.

[60] Rondina, Giacomo. 2008. "Incomplete Information and Informative Pricing."mimeo, Univerisity of California, San Diego.

[61] Runkle, David E. 1998. "Revisionist History: How Data Revisions Distort Economic Policy Research." *Federal Reserve Bank of Minneapolis Quarterly Review* No. 2241.

[62] Sims, Christopher A. 2003. "Implications of Rational Inattention." *Journal of Monetary Economics*, 50(3): 665-690.

[63] Sims, Christopher A. and Tao Zha. 1999. "Error Bands for Impulse Responses." *Econometrica*, 67(5): 1113-55.

[64] Svensson, Lars E.O. and Michael Woodford. 2003. "Indicator Variables for Optimal Policy." *Journal of Monetary Economics*, 50(3): 691-720.

[65] Svensson, Lars E.O. and Michael Woodford, 2005, "Indicator Variables for Optimal Policy under Asymmetric Information." *Journal of Economic Dynamics and Control.*

[66] Tambalotti, Andrea. 2003. "Optimal Monetary Policy and Productivity Growth." mimeo, Federal Reserve Bank of New York.

[67] Townsend, Robert. 1983. "Forecasting the Forecasts of Others." *Journal of Political Economy*, 91(4): 546-587.

[68] Veldkamp, Laura and Stijn Van Nieuwerburgh. 2006. "Learning Asymmetries in Real Business Cycles." *Journal of Monetary Economics*, 53(4): 753-772.

[69] Woodford, Michael. 2002. "Imperfect Common Knowledge and the Effects of Monetary Policy." in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press.

[70] Woodford, Michael. 2003. *Interest and Prices*. Princeton: Princeton University Press.